

Maximum Cut Vertices

MCS-236

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The path P_n has $n - 2$ cut vertices. We can show that this is the most cut vertices for any graph of order n .

Lemma 1 *If T is a spanning tree of a nontrivial connected graph G , then T has at least as many cut vertices as G does.*

Proof. Any cut vertex of G is a cut vertex of T . because for any three vertices u , v , and w , if all paths from u to w in G pass through v , then the same must be true in T . ■

Theorem 1 *If G is a nontrivial connected graph of order n , then G has at most $n - 2$ cut vertices.*

Proof. Any tree of order n has at least two vertices that are not cut vertices, namely the leaves. Therefore, any spanning tree T of G has at most $n - 2$ cut vertices. By Lemma 1, G has no more cut vertices than T does, so G too has at most $n - 2$ cut vertices. ■