Test 4 Topics
MCS115 - Fall 2008

Section 5.1, 5.2:

- You should be familiar with the idea of "rubber sheet geometry."
- You should understand what it means for two objects to be equivalent by distortion. What is allowed and what is not allowed when carrying out a distortion.
- How to show two objects are equivalent by distortion (by drawing pictures or describing the distortion).
- How to show things are not equivalent by distortion (usually by finding a property that would be preserved by distortions that one has and the other does not). Why a circle and a circle with a tail are not equivalent. How to classify letters of alphabet. Why a sphere is not equivalent to a torus. Removing points or cutting circles and into how many pieces resulting object is divided.
- How to tell whether tori with various numbers of "holes" are equivalent by distortion. (Counting holes as well as number of nonseparating cuts.)
- Some important shapes: sphere, torus, Möbius band, Klein bottle
- Determine number of edges and number of sides of a surface.
- Edge identification diagrams (rectangles with labeled edges (or arrows) that represent 2-dimensional surfaces)
  - How they work.
  - How to make them for the Möbius band, torus, Klein bottle
  - How to represent 2-dimensional surfaces with labeled rectangles.
- What happens when you cut a Möbius band? What happens when you cut a loop with two half twists?
- What happens when you cut a Klein bottle?

Section 5.3:

- You should know what a connected graph in the plane is.
- Euler's formula: $V - E + F = 2$
  - What it means
  - When it applies
  - When it doesn't apply
  - How to prove it (adding an edge means adding a vertex or adding a region)
  - How to use it to help you "count" faces, edges, vertices of a solid

Section 6.1:

- You should be able to recognize self-similarity in fractals including the Mandelbrot set, the Julia set, Sierpinski's triangle, Barnsley's fern, etc.

Section 6.2:

- You should understand what an iterative process is (see Repeat, Repeat, Repeat on pages 413-414).
- You should be familiar with simple compounding interest problems.
- You should know the rules of Conway's Game of Life. You should be able to determine several generations given a starting population. You should be able to recognize a population explosion, an extinction, a stable pattern, a periodic pattern, or a migratory pattern. You should be able to determine whether a given starting population gives rise to an extinction, a stable pattern or a periodic pattern.
- You should be able to explain what is meant by "sensitivity to initial conditions." You should understand that sensitivity to initial conditions is an important characteristic of chaos.

Section 6.3:

- You should understand the collage method creating repeated images and that it creates fractals (see page 438).
- You should be familiar with the Koch curve and Sierpinski's triangle/gasket.
- You should know that if one repeatedly applies a specific set of collage making instructions, starting with any initial image, the infinite collage process will produce the same result.
- Given a set of instructions, you should be able to construct several stages in the process.
- You should be able to play the Chaos Game. You should know that if you play the Chaos Game many times, you will generate a Sierpinski Triangle. You should understand the connection between the rules of the Chaos Game and the structure of the Sierpinski Triangle, i.e. why does one generate the other?
- Given a sub-triangle in Sierpinski's triangle, you should be able to devise a series of moves in the Chaos Game which would land you in that sub-triangle.