MCS 118 – Calc/Precalc I

Optimization problems:

1. Find the maximum and minimum values (if they exist) of each function on the given interval. Be sure to explain why your answer is correct.

   (a) $f(x) = x^3 - 3x^2 + 4$ on $[0, 4]$

   (b) $g(x) = 30 + 2x + \frac{72}{x}$ on $(0, \infty)$

2. The sum of two positive numbers is 48. What is the smallest possible value for the sum of their squares?

   (a) Pick two letters to represent the numbers. Write an equation that expresses the fact that the sum of the numbers is 48. This is your fixed equation.

   (b) Now write a formula that gives the sum of their squares. This is your optimizing formula.

   (c) Solve your fixed equation for your favorite letter and substitute in to the optimizing formula. This makes your optimizing formula into a function of one variable.

   (d) Next you need to figure out what interval makes sense. What is the smallest value that the one variable could be? What is the largest value? (You may want to look back at the fixed equation to figure out what the biggest value is.)

   (e) Now find the minimum value of your optimizing function on that interval.

3. Barbara needs to build a rectangular storage container for all her yarn. She needs 10 cubic meters of storage space, and the container does not need to have a top. The base of the container must be twice as long as it is wide. Material to make the base costs \$10 per square meter and material for the sides costs \$6 per square meter. Find the cost of materials for the cheapest possible container.

   (a) What quantity is fixed here? What can vary?

   (b) Draw a picture of the container and label the width, length, and height. Use the information given to express two of these in terms of the third.

   (c) Find the optimizing function.

   (d) What interval makes sense for this problem?

   (e) Now find the dimensions that will minimize the cost of the box.

4. Tom is planning to plant a small orchard, and is gathering information about the amount of fruit he can expect to harvest each year once the trees mature. He discovers that if he plants up to 60 trees of a particular type on the plot of land, the average harvest from each tree will be about 120 kg, but for each additional tree planted the expected yield will go down by an average of 2 kg per tree as a result of overcrowding. Naturally, he wants to plant for the maximum yield of fruit. How many trees should he plant?

   (a) This problem is slightly different than the other two because there is nothing that is fixed. Instead, the total yield of fruit will be a function of the number of trees.
   What will the yield be if there are 60 or fewer trees?

   (b) What will the yield be if Tom plants 75 trees? What if he plants 100 trees? What will the yield be if he plants more than 60 trees?

   (c) Find the optimizing function. Note that it is a piecewise-defined function.

   (d) What interval makes sense for this problem?

   (e) What are the critical points of this function?

   (f) How many trees should Tom plant in order to maximize his yield?