Worksheet on Related Rates (Section 2.7)

Steps for Solving Related Rates Problems

1. Make a drawing of the situation if possible.
2. Use letters to represent the variables involved in the situation – say x, y.
3. Identify all rates of change given and those to be determined. Use calculus notation $\frac{dx}{dt}, \frac{dy}{dt}$, etc, to represent them.
4. Determine an equation that both
    a. Involves the variables of step 2 and
    b. Will involve the derivative of step 3, when differentiated

    (You may need some geometrical formulas to do this)
5. Differentiate (by implicit differentiation) the equation of step 4
6. Substitute all known values into the differentiated equation
7. Use algebraic manipulation, if necessary, to solve for the desired unknown rate or quantity.

Example 1. The radius of a spherical balloon is increasing by 2 cm/sec. At what rate is air being blown into the balloon at the moment when the radius is 10 cm? Give unit in your answer.

Example 2

A kite is flying 150 m high, where the wind causes it to move horizontally at the rate of 5 m per second. In order to maintain the kite at a height of 150 m , the person must allow more string to be let out. At what rate is the string being let out when the length of the string already out is 250 m?

Example 3

(a) Water runs into a inverted conical tank at the rate of 7 cubic feet per minute. The radius of the water's surface is always half the height of the water. How fast is the water level rising when the water is 2 feet deep?

(b) Suppose water is leaking out of the tank at a rate of 2 feet/sec in (a). How fast is the radius of surface of the water increasing when the water is 2 feet deep?

Example 4
A lighthouse is on a small island 3 km away from the nearest point P on a straight shoreline and its light makes 4 revolutions per minute. How fast is the beam of light moving along the shoreline when it is 1 km from P?

Example 5
A 1.6 m tall woman is walking away from a street light which is at the top of a 5 m pole with a speed of 2 m/sec along a straight path.

(a) How fast is the tip of her shadow moving when she is 15m from the pole?
(b) How fast is her shadow lengthening at that point?