Linear Difference Equations:
Tables, Graphs, and Formulas

Launch Maple.
In the pop-up window, click on **`Start with a blank worksheet`**.

Reminders

You are encouraged to click on **Help** for each of the commands introduced below.
If at any time you lose the > prompt you need for command entry,
click Insert > Execution Group > Before Cursor/After Cursor.
You may enter commands in Math mode (the default) or Text mode.
In **Math mode**, the math notation you enter from the keyboard will be redisplayed on the screen so that fractions, exponents, etc. look as they do in math books.
In Math mode, you'll need to press the right arrow to get out of an exponent or denominator.
I have entered some commands in **Text mode** so that you can better see the keystrokes.
Commands may be concluded with a semicolon (;) or a colon (:) or, in Math mode, nothing.
Press Enter with the cursor anywhere in the command line to execute the command.
Note: A colon (:) at the end of a command suppresses output.
In most cases you should use a semicolon (;) or just press Enter in Math mode.
To continue a command on a new line, press Shift-Enter.
To add **comments** as I have in this worksheet, select Insert > Text.

PART 1: LESSON/DEMONSTRATION

Let's solve $y(k+2) + 0.3y(k+1) - 0.4y(k) = 2$
with initial conditions $y(0) = 1$ and $y(1) = -1$.
First rewrite it in the form $y(n) = ...$
The following Maple procedure incorporates this.

```
> y := proc(n) option remember;
    if n = 0 then 1 else if n = 1 then -1 else 2 - 0.3*y(n - 1) + 0.4*y(n - 2) end if end if end
      proc:
```

TABLE OF VALUES

I get a table using **evalm** (evaluate matrix) and **seq** (sequence).
Here [n, y(n)] denotes an ordered pair of values.

```
> evalm(matrix(26,2,[seq([n,y(n)],n=0..25)]));
```

GRAPH

Next let's get a graph. I need to load the **plots** package to use **listplot**.

```
> with(plots): listplot([seq([n,y(n)],n=0..25)]);
```

SOLUTION: FUNCTIONAL FORMULA

Would you like to get Maple to solve the difference equation for you,
giving a formula for $y(n)$?
Maple's **rsolve** command can solve recurrences.
I got an error when I tried to use "y" for the name of the sequence below,
perhaps owing to a conflict with the procedure definition of y,
so I changed the name to $y1$.

```
> rsolve({y1(n) = 2 - 0.3*y1(n - 1) + 0.4*y1(n - 2), y1(0) = 1, y1(1) = -1}, y1(n));
```

Isn't that slick?!

PART 2: APPLICATION: ECON MODEL

Samuelson's Model

```
> Y(t) = alpha·(1 + beta)·Y(t - 1) - alpha·beta·Y(t - 2) + 1
```

Implement this with alpha = 0.5 and beta = 1.

```
> Y := proc(t) option remember;
    if t = 1 then 2 else if t = 2 then 3 else 0.5·(1 + 1)·Y(t - 1) - 0.5·1·Y(t - 2) + 1 end if end
      if end proc:
```

Have Maple make a table of Y(t) for t = 1, 2, 3, ... 25.

Have Maple make a graph of Y(t) versus t.

Sketch your graph below

If I want to clear the memory for a fresh start, I can use the **restart** command.

```
> restart
```

NOTE: If I have Maple solve this difference equation, I get a solution involving complex numbers. In the Maple output, "I" denotes the square root of -1.
I have not figured out how to get Maple to convert this answer to the form Goldberg gives on p. 142, in terms of a cosine function. If you do, *please tell me how you do it*!

```
> rsolve( {y(t) = 1·y(t - 1) - 0.5·y(t - 2) + 1, y(1) = 2, y(2) = 3}, y(t) )
```

$$(-1+\mathrm{I})\left(\frac{1}{2}-\frac{1}{2}\,\mathrm{I}\right)^{t}+(-1-\mathrm{I})\left(\frac{1}{2}+\frac{1}{2}\,\mathrm{I}\right)^{t}+2 \qquad \textbf{(1)}$$

Extra credit: Repeat for alpha = 0.8 and beta = 2.

Sketch your graph(s) here: