LINEAR OPERATORS

A **linear operator L** obeys these rules: L(c u(k)) = c L(u(k)) for an arbitrary constant c and L(u(k)+v(k)) = L(u(k)) + L(v(k)) for arbitrary functions/sequences u(k) and v(k).
These two rules may be combined into one:
L(a u(k) + b v(k)) = a L(u(k)) + b L(v(k)) for all constants a and b and all functions/sequences u(k) and v(k).

Let L(u(k)) = u(k+1) - u(k)

k	u(k)	5 u(k)	L(u(k))	L(5u(k))	5 L(u(k))
0	0	0	1	5	5
1	1	5			
2	4				
3	9				
4	16				

k	u(k)	v(k)	u(k)+v(k)	L(u(k))	L(v(k))	L(u(k)+v(k))	L(u(k))+L(v(k))
0	0	0	0	1	2	3	3
1	1	2	3				
2	4	4					
3	9	6					
4	16	8					

Check **algebraically** that L, defined as the forward difference operator, is a linear operator.

Now let L(y(k)) = y(k+2) - 5y(k+1) + 6y(k), the left-hand side of equation (3.20) in Goldberg's text.

k	u(k)	5 u(k)	L(u(k))	L(5u(k))	5 L(u(k))
0	0	0	-1	-5	
1	1	5			
2	4	20			
3	9				
4	16				

k	u(k)	v(k)	u(k)+v(k)	L(u(k))	L(v(k))	L(u(k)+v(k))	L(u(k))+L(v(k))
0	0	0	0	-1	-2	-7	
1	1	2	3				
2	4	4	8				
3	9	6					
4	16	8					

Check **algebraically** that L, defined as above, is a linear operator.

Let **L** be the linear operator **L**(y(k)) = y(k+2) - 5 y(k+1) + 6 y(k).

k	$u(k)=2^k$	L(u(k))	$v(k)=3^k$	L(v(k))
0	1		1	
1	2		3	
2	4		9	
3	8		27	
4	16		81	

Check **algebraically** that L(u(k)) = 0 if $u(k) = 2^k$ and L(v(k)) = 0 if $v(k) = 3^k$.

Assume that $y(k) = r^k$ for some constant r.

Calculate (algebraically) $L(r^k)$.

Determine the (quadratic) equation that r must satisfy if we require $L(r^k) = 0$.

Solve this quadratic equation.

Think about what this worksheet has illustrated. Write down your observations and insights.

Again consider the linear operator L(y(k)) = y(k+2)-5y(k+1)+6y(k).
The equation L(y(k)) = 0 is the same as
the homogeneous LDEwcc y(k+2)-5y(k+1)+6y(k) = 0.
This can be written as y(n) = 5y(n-1) - 6y(n-2).
Given the initial conditions y(0)=0 and y(1)=1,
calculate y(n) for n=2, 3, …, 6. Also tabulate the values of 2^n and 3^n.

n	y(n)	2^n	3^n	
0	0			
1	1			
2				
3				
4				
5				
6				

Figure out values of constants A and B that make
$A\ 2^n + B\ 3^n = y(n)$.

A =			B=	

Calculate the values of A*2^n+B*3^n for n=0, 1, 2, …, 6
in the last column above and make sure they're the same as y(n) values.