Geometric Sums and Mixed Models

Refer to chapter 12 of [1], "Geometric Sums and Mixed Models" and section 2.4 of [2], "The equation $y_{k+1} = Ay_k + B$."

1. Calculate the values of the following sums [1, p. 280 #1]:

   (a) $1 + 1/2 + 1/4 + 1/8 + \cdots + (1/2)^{10}$

   (b) $1 + 1.2 + 1.2^2 + \cdots + 1.2^{20}$

   (c) $\sum_{k=0}^{15} 2^k$

   (d) $\sum_{k=0}^{10} 9 \cdot .1^k$

2. "A mixed-model difference equation is defined by $a_{n+1} = 1.3a_n + 2.4$; $a_0 = 5$. Work out a pattern for a_n, and use it to develop a functional equation for a_n." [1, p. 280 #2] Alternatively, use the formula for the solution of this mixed model.

3. "Suppose we are given the difference equation

$$y_{k+1} = 2y_k + 1 \qquad k = 0, 1, 2, \cdots$$

   with the initial condition $y_0 = 5$." [2, p. 65]

   (a) Complete the following table:

k	0	1	2	3	4
y_k					

(b) Write a formula for y_k as a function of k. Put it in simplified form. Check that it gives you the same values for y_k as the table did.

4. (a) Solve the difference equation [2, p. 67, #2(e)] with $y_0 = 2$:

$$2y_{k+1} + y_k - 3 = 0.$$

   (b) Calculate the first few values of y_k from your functional equation solution.

   (c) Compare them with the first few values of y_k calculated directly from the difference equation.

References

[1] Kalman, Dan, *Elementary Mathematical Models: Order Aplenty and a Glimpse of Chaos*, The Mathematical Association of America, 1997.

[2] Goldberg, Samuel, *Introduction to Difference Equations: with Illustrative Examples from Economics, Psychology, and Sociology*, Dover Publications, New York, 1958, 1986.