Trigonometric Functions and Their Derivatives

The tangent, cotangent, secant, and cosecant functions may be expressed in terms of the sine and cosine functions:

\[ \tan x = \frac{\sin x}{\cos x} \quad \cot x = \frac{\cos x}{\sin x} \]

\[ \sec x = \frac{1}{\cos x} \quad \csc x = \frac{1}{\sin x} \]

Notice that for simple inputs like \( x \) we write \( \sin x \) instead of \( \sin(x) \) (“sine of \( x \)”), etc. We also write \( \sin^2 x \) for \( (\sin(x))^2 \), etc.

The derivatives of these functions may be obtained from the derivatives of the sine and cosine functions by using the quotient rule and the trigonometric identity

\[ \sin^2 x + \cos^2 x = 1. \]

Incidentally, if we divide this identity by \( \cos^2 x \) and by \( \sin^2 x \), we get the following sometimes handy identities:

\[ \tan^2 x + 1 = \sec^2 x. \]

\[ 1 + \cot^2 x = \csc^2 x. \]

Here are the derivatives of all six trigonometric functions:

\[ \frac{d}{dx} \sin x = \cos x. \quad \frac{d}{dx} \cos x = -\sin x. \]

\[ \frac{d}{dx} \tan x = \sec^2 x = \frac{1}{\cos^2 x}. \quad \frac{d}{dx} \cot x = -\csc^2 x. \]

\[ \frac{d}{dx} \sec x = \sec x \tan x. \quad \frac{d}{dx} \csc x = -\csc x \cot x. \]

Learn them. Notice that the derivative formulas for the co-functions may be obtained from the derivative formulas for the functions (sine, tangent, secant) by replacing each function by its co-function and putting in a minus sign. (The “co-cosine” is the sine.)