Trigonometric Functions and Their Derivatives

The tangent, cotangent, secant, and cosecant functions may be expressed in terms of the sine and cosine functions:

$$\tan x = \frac{\sin x}{\cos x}. \qquad \cot x = \frac{\cos x}{\sin x}.$$

$$\sec x = \frac{1}{\cos x}. \qquad \csc x = \frac{1}{\sin x}.$$

Notice that for simple inputs like x we write $\sin x$ instead of $\sin(x)$ ("sine *of* x"), etc. We also write $\sin^2 x$ for $(\sin(x))^2$, etc.

The derivatives of these functions may be obtained from the derivatives of the sine and cosine functions by using the quotient rule and the trigonometric identity

$$\sin^2 x + \cos^2 x = 1.$$

Incidentally, if we divide this identity by $\cos^2 x$ and by $\sin^2 x$, we get the following sometimes handy identities:

$$\tan^2 x + 1 = \sec^2 x.$$

$$1 + \cot^2 x = \csc^2 x.$$

Here are the derivatives of all six trigonometric functions:

$$\frac{d}{dx}\sin x = \cos x. \qquad \frac{d}{dx}\cos x = -\sin x.$$

$$\frac{d}{dx}\tan x = \sec^2 x = \frac{1}{\cos^2 x}. \qquad \frac{d}{dx}\cot x = -\csc^2 x.$$

$$\frac{d}{dx}\sec x = \sec x \tan x. \qquad \frac{d}{dx}\csc x = -\csc x \cot x.$$

Learn them. Notice that the derivative formulas for the co-functions may be obtained from the derivative formulas for the functions (sine, tangent, secant) by replacing each function by its co-function and putting in a minus sign. (The "co-cosine" is the sine.)