# MCS 121 – Calculus I

Indeterminate Forms and L'Hospital's Rule

Evaluate the following limits:

1. $\lim_{t\to 0} \dfrac{\sin(t^2)}{t^2}$

2. $\lim_{x\to 0} \dfrac{\sin x}{1+\sin x}$

3. $\lim_{x\to 0} \dfrac{3^{\sin x}-1}{x}$

4. $\lim_{x\to \infty} \dfrac{\ln x}{\sqrt{x}}$

5. $\lim_{x\to 0} \dfrac{\cos x - 1 + \frac{1}{2}x^2}{x^4}$

6. Write a ratio of two functions such that in order to calculate the limit one must apply L'Hôpital's rule three times. Now make it six times.

Answers: $1, 0, \ln 3, 0, 1/24, \lim_{x\to 0}(\sin x - x)/x^3, \lim_{x\to\infty} x^6/e^x$

1. $\lim_{x \to 1} \dfrac{x^2 + 3x - 4}{x - 1}$

2. $\lim_{x \to 0} \dfrac{e^x - 1}{\sin x}$

3. $\lim_{x \to \infty} \dfrac{(\ln x)^3}{x^2}$

4. $\lim_{x \to \infty} x^3 e^{-x^2}$

5. $\lim_{x \to 0} \left( \dfrac{1}{x^4} - \dfrac{1}{x^2} \right)$

6. $\lim_{x \to 0} \dfrac{\sin x}{1 - \cos x}$

7. $\lim_{x \to 0} \dfrac{\tan x}{x}$

8. $\lim_{x \to \infty} \left( x - \sqrt{x^2 - 1} \right)$

9. $\lim_{x \to 1} \left( \dfrac{1}{\ln x} - \dfrac{1}{x - 1} \right)$

Answers: $5, 1, 0, 0, \infty$, does not exist, $1, 0, 1/2$

Sometimes you can handle indeterminate forms like "$0^0$", "$\infty^0$", "$0^\infty$', and "$1^\infty$" by taking the log, finding the limit, and then exponentiating to get the answer. The following limits are mostly of this type.

1. $\lim_{x\to 0+} x^x$

2. $\lim_{x\to 0+} x^{1/\ln x}$

3. $\lim_{x\to 0+} (1+x)^{1/x}$

4. $\lim_{x\to\infty} \left(1+\frac{1}{x}\right)^x$

5. $\lim_{x\to\infty} \frac{x^n}{e^{kx}}$ where $k > 0$

6. $\lim_{x\to\infty} \frac{x^n}{\ln x}$ if $n > 0$

7. $\lim_{x\to\pi/2} (\tan x)^{x-\pi/2}$

Answers: $1, e, e, e, 0, \infty, 1$