Linear Approximation and the Derivative

1. Let $f(x) = \sqrt{x}$.

   (a) Find the tangent line approximation to (local linearization of) $f(x) = \sqrt{x}$ near $x = 4$.

   (b) $\sqrt{4.2} \approx ?$

   (c) Is your approximation of $\sqrt{4.2}$ an overestimate? Or an underestimate? Determine the answer without evaluating $\sqrt{4.2}$ more accurately.

2. Sect. 3.9 #4, p. 154:
   "Show that $1 - x/2$ is the tangent line approximation to $1/\sqrt{1+x}$ near $x = 0$."

Theorems about Differentiable Functions

1. Is every continuous function also differentiable?

2. Is every differentiable function also continuous?

3. Is it possible to define a function $f(x)$ that is continuous for $a < x < b$ but $f'(x)$ does not exist at *every* x between a and b?

4. **MVT: Mean Value Theorem**

   (a) State (i) the hypothesis of the Mean Value Theorem and (ii) the conclusion of the Mean Value Theorem. Also identify the main verb.

   (b) Sect. 3.10 #8-9, p. 158:
   "Do the functions graphed ... appear to satisfy the hypotheses of the Mean Value Theorem on $[a, b]$?

   (c) Draw a large rectangle divided into quadrants. Label the left side "Hypothesis." Label the top side "Conclusion." Label the two "rows" on the left side "True" and "False." Label the two "columns" on the top side "True" and "False." Then in each of the quadrants draw a graph, if you can, that illustrates each case. Thus, the upper left quadrant would show the graph of a function for which both the hypothesis and conclusion of the MVT are true.

5. Sect. 3.10 #11, p. 158:
   "Use the **Racetrack Principle** [or another theorem in this section] to show that $\ln x \leq x - 1$" for $x > 0$. Note: This problem naturally breaks into two parts: $x \geq 1$ and $0 < x \leq 1$.