Fall 2007 Calculus I Final Exam Answers

1. (a) $y' = \frac{15}{2}x^{1/4} - \frac{3}{4x^2} - 6x^2$.

   (b) $y' = \sec^2(x) \cdot \arcsin(x) + \frac{\tan(x)}{\sqrt{1-x^2}}$.

   (c) $y' = \frac{2x^2+2x-8}{(2x+1)^2}$

   (d) $y' = \frac{-3e^{3x}}{2(25+e^{3x})^{3/2}}$.

   (e) $y' = \frac{4x^3}{\cos(y) - y\sin(y)}$.

2. (a) The derivative of g at x is the value of $\lim_{h\to 0} \frac{g(x+h)-g(x)}{h}$ if this limit exists.

   (b) $g'(x) = \lim_{h\to 0} \frac{[6(x+h)-(x+h)^2]-[6x-x^2]}{h} = \lim_{h\to 0} \frac{6x+6h-x^2-2xh-h^2-6x+x^2}{h} = \lim_{h\to 0} \frac{6h-2xh-h^2}{h} = \lim_{h\to 0}(6-2x-h) = 6-2x$.

   (c) $y = -2x + 16$.

3. (a) $\lim_{x\to\infty} \frac{2e^x+5}{3e^x+7} = \frac{2}{3}$.

   (b) $\lim_{x\to 3} \frac{3x-3}{3x^2-3} = \frac{1}{4}$.

   (c) $\lim_{x\to 0} \frac{\cos(x)-1}{7x^2} = \frac{-1}{14}$.

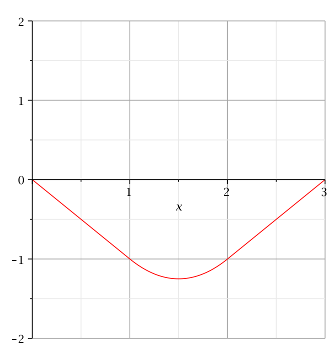
4. (a) (Graph not shown)

   (b) Upper sum $\doteq 0.4040$; lower sum $\doteq 0.3920$.

   (c) Estimated value $= 0.3980 \pm 0.0060$.

5. (a)

t	0	1	1.5	2	3
F(t)	0	−1	−1.25	−1	0



   (b)

6. (a) Cite the Second Fundamental Theorem of Calculus and the chain rule.

(b) $x = 0$ only

(c) $F''(x) = 16xe^{-2x-1} - 16x^2e^{-2x-1} = 16xe^{-2x-1}(1-x)$.

(d) $x = 0$ and $x = 1$.

7. (a) $\displaystyle\int \frac{1+y^2}{y}\,dy = \ln|y| + (1/2)y^2 + C$.

   (b) $\displaystyle\int_0^{\pi} \sin(\alpha)\,d\alpha = 2$.

   (c) $\displaystyle\int \left(x^5 + \frac{2}{x} + \frac{1}{\sqrt[4]{x^2}}\right) dx = \frac{x^6}{6} + 2\ln|x| + 2x^{1/2} + C$.

   (d) $\displaystyle\int_{-1}^{1} (2x+3)\,dx = 6$.

8. (a) $v(t) = -1.6t + 16$ m/s.

   (b) Height $= -0.8t^2 + 16t + 2$ meters.

   (c) 82 m (when $t = 10$).

9. Volume $V = \pi r^2 h = 3920\pi$, so $h = 3920/r^2$, and cost $C = 7 \cdot 2\pi r h + 10\pi r^2 = 54880\pi/r^2 + 10\pi r^2$. Cost is minimized when radius $r = 14$ feet and height $= 20$ feet. $d^2C/dr^2 = (2)(54880\pi)/r^3 + 20\pi > 0$ for $r > 0$, so C is minimum where $dC/dr = 0$.

10. (a) $F'(x) = 1 \cdot \sin x + x \cdot \cos x - \sin x = x\cos x$.

    (b) Area $= F(\pi/2) - F(0) = \pi/2 - 1$.