MCS121 Final Exam, Fall 2007

This test is a closed-book test; you are allowed an $8\frac{1}{2}$–by–11–inch crib sheet (or 3 small note cards) and a graphing calculator. Please write your name below. Look at all of the problems before deciding which to do first. Note that some problems are easier than others, and some are worth more points. In order to earn full credit, your solution steps must be clearly presented. Be sure to use notation correctly.

You may use the back of the pages if you need additional space, and you may request additional paper. You have two hours to work.

Name: ______________________________

Instructor: (circle one)

Glubokov at 09:00	Holte at 12:30	Rietz at 8:00
Glubokov at 11:30	Holte at 2:30	Rietz at 10:30
Glubokov at 1:30		

Problem	Page	Possible	Score
1	2	25	
2	3	20	
3	4	15	
4	5	20	
5	6	20	
6	7	20	
7	8	24	
8	9	16	
9	10	25	
10	11	15	
Total		200	

Formulas for volume V and surface area A:

Sphere: $V = \frac{4}{3}\pi r^3$ $\qquad A = 4\pi r^2$

Cylinder: $V = \pi r^2 h$ $\qquad A = 2\pi r h + 2\pi r^2$

Cone: $V = \frac{1}{3}\pi r^2 h$ $\qquad A = \pi r\sqrt{r^2 + h^2} + \pi r^2$

1. [**25 Points**] Find the derivative ($y' = \frac{dy}{dx}$) for each of the following functions. Give appropriately simplified answers.

(a) $y = 6\sqrt[4]{x^5} + \dfrac{3}{4x} - 2x^3$

(b) $y = \tan(x)\arcsin(x)$

(c) $y = \dfrac{x^2+4}{2x+1}$

(d) $y = \dfrac{1}{\sqrt{25+e^{3x}}}$

(e) (Implicit differentiation) $x^4 + 3 = y\cos(y)$

2. [**20 Points**]

(a) State the formal limit definition of the derivative of the function g at the point x.

(b) Let $g(x) = 6x - x^2$. Use the formal limit definition of the derivative to show that $g'(x) = 6 - 2x$. (You will not get credit for using a short-cut derivative formula to find $g'(x)$.)

(c) Find an equation of the tangent line to the graph of $y = 6x - x^2$ at $x = 4$.

3. [**15 Points**]

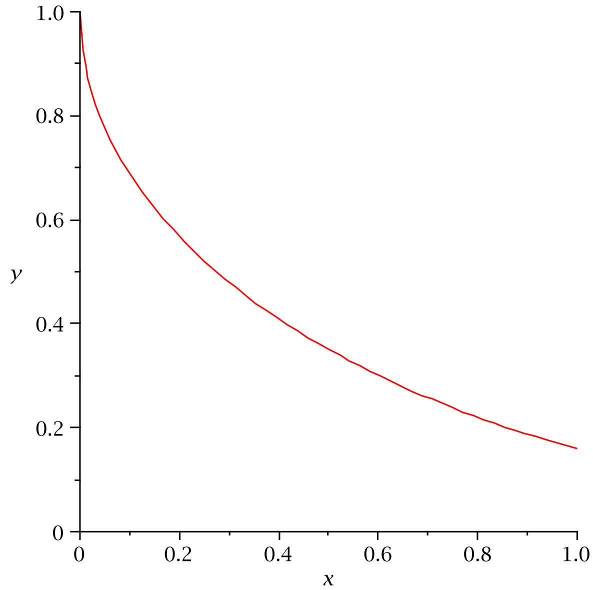
Evaluate the following limits.

(a) $\lim_{x\to\infty} \frac{2e^x + 5}{3e^x + 7}$

(b) $\lim_{x\to 3} \frac{3x - 3}{3x^2 - 3}$

(c) $\lim_{x\to 0} \frac{\cos(x) - 1}{7x^2}$

4. [**20 Points**] The graph of $f(x) = 1 - \sin(\sqrt{x})$ on the interval $0 \le x \le 1$ is shown below.

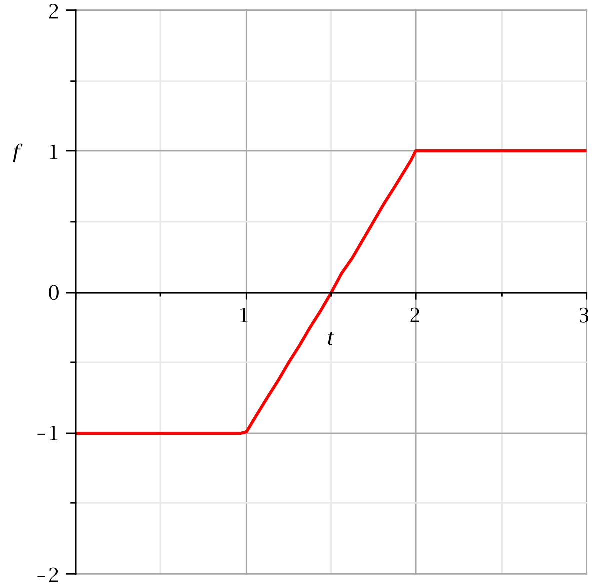


(a) Draw rectangles on the graph representing the left sum for $f(x)$ on the interval $0 \le x \le 1$ using 5 subdivisions.

(b) Using RSUMS with 70 subdivisions, find upper and lower estimates for $\int_0^1 f(x)\,dx$. Round your answers to four decimal places.

(c) Find a single number that estimates this integral, and give a bound on the error in your estimate.

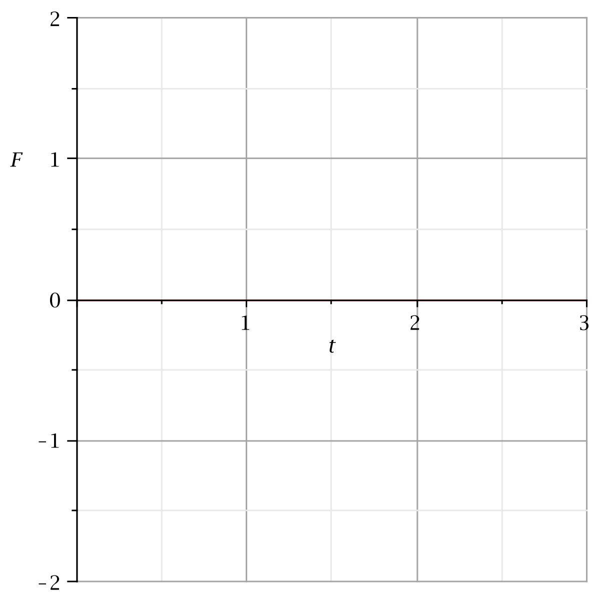
5. [**20 Points**] Below is the graph of a function $f(t)$ from $t = 0$ to $t = 3$. Let $F(t)$ be a function such that $\frac{dF}{dt} = F'(t) = f(t)$. Suppose $F(0) = 0$.



(a) Fill in the table of values:

t	0	1	1.5	2	3
F(t)					

(b) Sketch a graph of $F(t)$:



6. [**20 Points**] Let a function $F(x)$ be defined for all x by

$$F(x) = \int_1^{2x+1} (t-1)^2 \, e^{-t} \, dt.$$

(a) Verify that that the derivative of $F(x)$ is given by $F'(x) = 8x^2 \, e^{-2x-1}$, and cite the theorems discussed in class that you use.

(b) Find all critical points for $F(x)$ and determine where the function is increasing and decreasing.

(c) Find the second derivative of $F(x)$.

(d) Find the x-values of the inflection points and where the graph of $F(x)$ is concave up and concave down.

7. [**24 Points**] Evaluate the following integrals analytically. As usual, show your work.

(a) $\displaystyle\int \frac{1+y^2}{y}\,dy$

(b) $\displaystyle\int_0^{\pi} \sin(\alpha)\,d\alpha$

(c) $\displaystyle\int \left(x^5 + \frac{2}{x} + \frac{1}{\sqrt[4]{x^2}}\right)\,dx$

(d) $\displaystyle\int_{-1}^{1} (2x+3)\,dx$

8. [**16 Points**] On the Moon the acceleration due to gravity is 1.6 meters per second per second (m/sec^2) downward. A cosmonaut throws a calculator up, releasing it from a height of 2 meters at a speed of 16 meters per second.

(a) Find a formula for $v(t)$, the velocity of the calculator t seconds after release.

(b) Find a formula for its height t seconds after release.

(c) What is the calculator's greatest height above the surface of the Moon?

9. [**25 Points**] A cylindrical container, with a base but no top, is to hold 3920π cubic feet. The material for the side costs \$7 per square foot, and the material for the base costs \$10 per square foot. What radius and height will minimize the total cost of material? Show that your answer gives a minimum.

10. [**15 Points**]

Let $F(x) = x\sin x + \cos x$.

(a) Show that $F(x)$ is an antiderivative of $f(x) = x\cos x$.

(b) Use the Fundamental Theorem of Calculus to calculate the exact area under the curve $y = x\cos x$ over the interval $0 \le x \le \pi/2$.

Honor pledge: Please consider signing the following honor pledge:

On my honor, I pledge that I have not given, received, or tolerated others' use of unauthorized aid in completing this work.

Name: ______________________________