MCS121
Answers to homework: 1.1, 1.2, 1.3, 1.4

1.1.3 (a) When the car is 5 years old, it is worth $6000.

(b) Since the value of the car decreases as the car gets older, this is a decreasing function. A possible graph:

(c) The vertical intercept is the value of \( V \) when \( a = 0 \), or the value of the car when it is new. The horizontal intercept is the value of \( a \) when \( V = 0 \), or the age of the car when it is worth nothing.

1.1.12 Parallel line: \( y - 5 = -4(x - 1) \) or \( y = -4x + 9 \)
Perpendicular line: \( y - 5 = .25(x - 1) \) or \( y = .25x + 4.75 \)
(The line \( y + 4x = 7 \) has slope \(-4\).)

1.1.14 (a) V (b) IV (c) I (d) VI (e) II (f) III

1.1.16 Domain: \( 0 \leq x \leq 5 \) or \([0, 5]\) Range: \( 0 \leq y \leq 4 \) or \([0, 4]\)

1.1.24 \( E = kv^3 \), for some constant \( k \).

1.1.29

1.1.30 (a) \( C = 12 + 0.2w \)

(b) The slope is 0.2 dollars per kilogram. Each additional kilogram of waste costs 20 cents.

(c) The intercept is 12 dollars. The flat monthly fee to subscribe to the waste collection service is $12. This is the amount charged even if there is no waste.

1.2.6 Initial quantity is 7.7; growth rate is \(-0.08 = -8\% \) (decay).
1.2.10  (a) This is a linear function with slope $-2$ grams per day and intercept $30$ grams. The function is $Q = -2t + 30$, and the graph is shown below.

(b) Since the quantity is decreasing by a constant percent change, this is an exponential function with base $1 - 0.12 = 0.88$. The function is $Q = 30(0.88)^t$, and the graph is shown above.

1.2.14  (a) $h(x)$ is linear; $h(x) = -3x + 31$  
(b) $g(x)$ is exponential; $g(x) = 36(1.5)^x$

1.2.16  (a) $125\%$; We have

Reduced size $= .80 \cdot$ Original size

or

Original size $= \frac{1}{.80}$Reduced size $= (1.25)$Reduced size,

so the copy must be enlarged by a factor of 1.25, which means it is enlarged to $125\%$ of the reduced size.

(b) If a page is copied $n$ times, then

New size $= (.80)^n \cdot$ Original size.

We want to solve for $n$ so that $(.80)^n = 0.15$. By trial and error, we find $(.80)^9 = 0.134$. So the page needs to be copied 9 times.

1.2.18  $y = 3(2^x)$

1.2.30  $f(x) = 5x + 10$ and $g(x) = 10(\sqrt{2})^x$

|x| \hline 0 & 1 & 2 & 3 & 4  
|---|---|---|---|---|---|
|f(x)| 10 & 15 & 20 & 25 & 30  
|g(x)| 10 & 10\sqrt{2} & 20 & 20\sqrt{2} & 40  

1.3.2  (a) $f(t + 1) = (t + 1)^2 + 1 = t^2 + 2t + 2$
(b) $f(t^2 + 1) = (t^2 + 1)^2 + 1 = t^4 + 2t^2 + 2$
(c) $f(2) = 5$
(d) $2f(t) = 2(t^2 + 1) = 2t^2 + 2$
(e) $(f(t))^2 + 1 = (t^2 + 1)^2 + 1 = t^4 + 2t^2 + 2$
1.3.6 \( m(z + h) - m(z) = (z + h)^2 - z^2 = 2zh + h^2 \)

1.3.10 (a) \( f(10,000) \) represents the value of \( C \) corresponding to \( A = 10,000 \), or in other words the cost of building a 10,000 square-foot store.

(b) \( f^{-1}(20,000) \) represents the value of \( A \) corresponding to \( C = 20,000 \), or in other words the area in square feet of a store which would cost \$20,000\) to build.

1.3.14 The function is not invertible. It fails the horizontal line test.

1.3.26 \( g(f(2)) \approx g(0.4) \approx 1.1.\)

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<th>( x )</th>
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<th>( g(x) )</th>
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1.3.36 \( x = \frac{\log(2/11)}{\log(7/5)} \approx -5.07 \)

1.4.14

\[
\begin{align*}
\ln(10^{x+3}) &= \ln(5e^{7-x}) \\
(x + 3)\ln 10 &= \ln(5 + (7 - x)\ln e) \\
\ln 10x + 3\ln 10 &= \ln 5 + 7 - x \\
\ln(10 + 1)x &= -3\ln 10 + \ln 5 + 7 \\
x &= (-3\ln 10 + \ln 5 + 7)/(\ln 10 + 1) \\
x &\approx 0.515 \\
\text{also } x &= (\log 5 + 7\log e - 3)/(1 + \log e)
\end{align*}
\]
1.4.36 We know that the \( y \)-intercept of the line is at \((0, 1)\), so we need one other point to determine the equation of the line. We observe that it intersects the graph of \( f(x) = 10^x \) at the point \( x = \log 2 \). The \( y \)-coordinate of this point is then \( y = 10^x = 10^{\log 2} = 2 \), so \((\log 2, 2)\) is the point of intersection. We can now find the slope and equation of the line. \( y = \frac{1}{\log 2}x + 1 \approx 3.3219x + 1 \).

1.4.38 (a) Since the initial amount of caffeine is 100mg and the exponential decay rate is 
\(-0.17\), we have \( A = 100e^{-0.17t} \).

(b) We estimate the half-life by estimating \( t \) when the caffeine is reduced by half (so \( A = 50 \)); this occurs at approximately \( t = 4 \) hours.

(c) We want to find the value of \( t \) when \( A = 50 \);
\[
50 = 100e^{-0.17t} \\
0.5 = e^{-0.17t} \\
\ln 0.5 = -0.17t \\
t = (\ln 0.5)/(-0.17) \approx 4.077.
\]

1.4.46 Let \( m \) be the infant mortality of Senegal. As a function of time \( t \) (in years), \( m \) is given by \( m = m_0(0.90)^t \). To find when \( m = 0.5m_0 \) (so the number of cases has been reduced by 50\%), we solve
\[
0.5m_0 = m_0(0.90)^t \\
0.5 = (0.90)^t \\
\ln(0.5) = t\ln(0.9) \\
t = \frac{\ln(0.5)}{\ln(0.9)} \approx 6.58 \text{ years}.
\]