MCS121
Answers to homework: 1.5, 1.6

1.5.20 This graph is an inverted cosine curve with amplitude 8 and period $20\pi$. One possibility is $f(x) = -8 \cos(\frac{x}{10})$. Other possibilities: $f(x) = 8 \sin(\frac{1}{10}(x - 5\pi))$, $f(x) = 8 \cos(\frac{1}{10}(x - 10\pi))$, or $f(x) = -8 \sin(\frac{1}{10}(x + 5\pi))$.

1.5.24 This graph is a sine curve with period 8, amplitude 3, and a vertical shift of 3. One possibility is $f(x) = 3 + 3 \sin(\frac{x}{7})$.

1.5.36 (a) $V_0$ represents the amplitude of the oscillation. It also represents the maximum voltage.
(b) The period is $\frac{2\pi}{12\pi} = \frac{1}{60}$ second.
(c) Since each oscillation takes 1/60 second, in 1 second there are 60 complete oscillations.

1.5.40 We use a cosine function of the form $H = A \cos(Bt) + C$. Since the period is 24 hours, $\frac{2\pi}{B} = 24$, giving $B = \frac{\pi}{12}$. The temperature oscillates around an average value of 60°F, so $C = 60$. The amplitude of the oscillation is 20°F. To arrange that the temperature be at its lowest when $t = 0$, we take $A$ negative. Thus $H = -20 \cos(\frac{\pi}{12}t) + 60$.

1.6.4 As $x \to \infty$, $y \to \infty$. As $x \to -\infty$, $y \to 0$.

1.6.6 (a) II and III because in both cases, the numerator and denominator each have $x^2$ as the highest power with coefficient 1. Therefore, $y \to \frac{x^2}{x^2} = 1$ as $x \to \pm \infty$.
(b) I, since $y \to \frac{x}{2x} = 0$ as $x \to \pm \infty$.
(c) II and III, since replacing $x$ by $-x$ leaves the graph of the function unchanged.
(d) None
(e) III, since the denominator is zero and $f(x)$ tends to $\pm \infty$ when $x = \pm 1$.

1.6.10 $f(x) = k(x + 3)(x - 1)(x - 4) = k(x^3 - 2x^2 - 11x + 12)$, where $k < 0$. ($k \approx -0.1$ if the horizontal and vertical scales are equal; otherwise one can’t tell how large $k$ is.)

1.6.22 (a) $R = kr^4$ where $k$ is a constant.
(b) Given $R = 400$ for $r = 3$, we can determine the constant $k$.

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\begin{align*}
400 &= k(3)^4 \\
400 &= k(81) \\
k &= \frac{400}{81} \approx 4.938.
\end{align*}
\]

So the formula is $R = 4.938r^4$.

(c) Evaluating the formula above at $r = 5$ yields $R = 4.928(5)^4 = 3086.42 \text{ cm}^3/\text{sec}$.