MCS 121 Derivative Skills Test, 31 October 2001

This is a closed-book test. You may **not** use any text or notes. Also calculators may **not** be used on this test. Make sure that you assert equalities appropriately and use notation correctly in expressing your answers.

Name: _______________________

The following trigonometric identities are given for reference.

\[
\tan x = \frac{\sin x}{\cos x}.
\]
\[
\cot x = \frac{\cos x}{\sin x}.
\]
\[
\sec x = \frac{1}{\cos x}.
\]
\[
\csc x = \frac{1}{\sin x}.
\]

<table>
<thead>
<tr>
<th>Problem</th>
<th>Page</th>
<th>Possible</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>4</td>
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<td>2</td>
<td>2</td>
<td>33</td>
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<td>3</td>
<td>35</td>
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<td>28</td>
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<tr>
<td><strong>Total</strong></td>
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<td><strong>100</strong></td>
<td></td>
</tr>
</tbody>
</table>

1. **[4 Points]** Let \( g \) be a function defined on an interval that includes the number \( x \). State the definition of the derivative of \( g \) at \( x \).
2. **[33 Points]** (3 points per part) Find the derivatives of the following functions. Express each answer in as simple a form as you can, using correct notation, of course.

(a) \( a(x) = \sin(x) \)

(b) \( b(x) = \tan(x) \)

(c) \( c(x) = \sec(x) \).

(d) \( d(t) = \cos(t) \)

(e) \( E(t) = \cot(t) \)

(f) \( f(u) = \csc(u) \)

(g) \( g(r) = 2^r \)

(h) \( y = x^e \)

(i) \( z = \log x \). Here \( \log x \) denotes \( \log_{10}(x) \).

(j) \( \theta = \arcsin y \)

(k) \( K(w) = \arctan w \)
3. [35 Points] (7 points per part) Find the derivatives of the following functions. You may leave your answers unsimplified. As usual, use correct notation.

(a) \( p(x) = 7x^4 - 6x^3 - 5x^2 + 1 \)

(b) \( y = \sqrt{x^7} - \frac{2}{\sqrt[3]{x^5}} + \frac{1}{x^4} \)

(c) \( u = e^{2x^3} \cos(4t) \)

(d) \( Q(t) = \sin^2(\ln(3t)) \)

(e) \( z = \frac{\arctan x}{1 + x^2} \)
4. [28 Points] (7 points per part) Find the derivative $y' = \frac{dy}{dx}$ in each case. Simplify your answers as much as possible. (Sometimes, perhaps, it may be to your advantage to simplify the function first.)

(a) $y = \frac{x^3 - 4x^2}{x^2 + 5}$

(b) $y = \frac{x^3}{4}(3\ln x - 1)$

(c) $y = \ln \left(\frac{1}{\cos x}\right)$

(d) $y = \tan(\arctan(x^3)) + \sec(x + y)$