

MCS 121–Fall 2004: Chapter 2 exam answers

- (a) Average rate of change = $(f(30) - f(20))/(30 - 20) = (88 - 20)/10 = 6.8$.

(b) $(f(20) - f(10))/(20 - 10) = (20 - 10)/(20 - 10) = 1$. $f'(20) \approx (6.8 + 1.0)/2 = 3.9$
(equivalently, $(f(30) - f(10))/(30 - 10) = (88 - 10)/10 = 3.9$).
- Any δ in $(0, .0399]$ will work. ($0 < \delta \leq 4\epsilon - \epsilon^2$.)
- (a) The statement “ $\lim_{x \rightarrow 2}(3x - 1) = 5$ ” means that for every positive ϵ there exists a positive δ such that (for every x) if $|x - 2| < \delta$ and $x \neq 2$ then $|(3x - 1) - 5| < \epsilon$.

(b) Let $\epsilon > 0$ be given. Let $\delta = \epsilon/3$. If $|x - 2| < \delta$ and $x \neq 2$, then $|(3x - 1) - 5| = |3x - 6| = 3|x - 2| < 3\delta = 3\epsilon/3 = \epsilon$, so $|(3x - 1) - 5| < \epsilon$. By definition of the limit (see (a)), $\lim_{x \rightarrow 2}(3x - 1) = 5$.
- a—2, b—3, c—1, d—6.
- (a)
$$f'(2) = \lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \rightarrow 0} \frac{[(2+h)^2 - 6(2+h)] - [(2)^2 - 6(2)]}{h}$$
$$= \lim_{h \rightarrow 0} \frac{4 + 4h + h^2 - 12 - 6h - 4 + 12}{h} = \lim_{h \rightarrow 0} \frac{-2h + h^2}{h} = \lim_{h \rightarrow 0} (-2 + h) = -2.$$

(b) The line through $(2, f(2)) = (2, -8)$ with slope -2 is $y - (-8) = -2(x - 2)$, or $y = -2x - 4$.
- Your graph of $y = f(x)$ should be increasing for $-3 < x < 0$, have a horizontal tangent line at $x = 0$, be decreasing for $0 < x < 2$, have a horizontal tangent line at $x = 2$, and be increasing for $2 < x < 4$.
- (a) Dollars, number of CDs, and number of CDs per dollar

(b) i. At a price of 15 dollars, the company can sell 3,600,000 CDs.
ii. If the price is 15 dollars, the instantaneous rate of change of the number of CDs sold is $-150,000$ CDs per dollar, so, practically, a small increase in the price (say \$1) will decrease the number of CDs sold by about 150,000 times the increase (a 150,000 sales decrease for a 1\$ increase).

(c) $f(15.5) \approx f(15) + f'(15)(0.5) = 3,600,000 - 75,000 = 3,525,000$.
- (a) $(-3, -1), (1, 3)$ (b) $[-3, -2), (0, 2)$ (c) $(-1, 1)$
- Varying table values are expected. The limit is $\ln 2 \approx 0.693$.
- (a) $x = 2$ (break in the graph) (b) $x = 2, -2$ (discontinuous, corner)