(1) \( \frac{dv}{dL} = \frac{1}{2} \left( \frac{L}{2} + \frac{2}{L} \right)^{-1/2} \left( \frac{1}{2} - \frac{2}{L^2} \right) \), so \( \frac{dv}{dL} = 0 \Leftrightarrow \frac{1}{2} = \frac{2}{L^2} \Leftrightarrow L^2 = 4. \) Since \( L > 0, \) \( L = 2. \)

This gives a minimum by the first derivative sign-change test: \( 0 < L < 2 \Rightarrow \frac{1}{2} - \frac{2}{L^2} < \frac{1}{2} - \frac{2}{2^2} = 0 \Rightarrow \frac{dv}{dL} < 0 \) and \( L > 2 \Rightarrow \frac{1}{2} - \frac{2}{L^2} > \frac{1}{2} - \frac{2}{2^2} = 0 \Rightarrow \frac{dv}{dL} > 0. \)

(Alternative tests: (1) \( v \to \infty \) as \( L \to 0^+ \) and as \( L \to \infty; \)
(2) \( \frac{d^2v}{dL^2} = \frac{2(\sqrt{2} + \frac{1}{L} - \frac{1}{2} - \frac{2}{L^2})(\frac{1}{L^2} + \frac{2}{L^2} - 1)^{1/2}(\frac{1}{L^2} - \frac{1}{L})}{4(\frac{1}{L^2} + \frac{2}{L^2})} = \ldots = \frac{\sqrt{2}}{8} > 0 \) at \( L = 2. \)
(3) Easier: minimize \( v^2. \) \( \frac{dv^2}{dL} = \frac{1}{2} - \frac{2}{L^2} = 0 \) at \( L = 2, \) and \( \frac{d^2v^2}{dL^2} = \frac{4}{L^3} > 0. \)

(2) \( f'(x) = \frac{1}{1 + x^2} \cdot 2x \) and \( f'(x) = 0 \) if and only if \( x = 0 \) (critical point). A global extreme must occur at an endpoint \( (x = -3, 2) \) or a critical point \( (x = 0): f(-3) = \ln 10, f(0) = \ln 1 = 0, \) and \( f(2) = \ln 5, \) so the global max occurs at \( x = -3 \) and the global min at \( x = 0. \)

(3) In the triangle given, let \( x \) be the length of the horizontal leg, and let \( z \) be the length of the hypotenuse. By the Pythagorean theorem, \( x^2 + 300^2 = z^2. \) Therefore, \( 2x \frac{dx}{dt} = 2z \frac{dz}{dt}. \) When \( z = 500, x = \sqrt{500^2 - 300^2} = 400 \) and \( \frac{dz}{dt} = 20, \) so \( 2 \cdot 400 \cdot \frac{dx}{dt} = 2 \cdot 500 \cdot 20, \) whence \( \frac{dx}{dt} = 25 \) ft/sec.

(4) (a) Use \( f(x) \approx f(a) + f'(a)(x-a) \) with \( f(x) = x^{1/4}, a = 1: x^{1/4} \approx 1^{1/4} + \frac{1}{4}(1)^{-3/4}(x-1) = 1 + \frac{1}{4}(x-1). \) (b) \( (1.1)^{1/4} \approx 1 + \frac{1}{4}(1.1) = 1.025. \) (c) \( \frac{d^2}{dx^2} x^{1/4} = -\frac{3}{16}x^{-7/4} < 0, \) so the graph is concave down and the tangent line approximation is an overestimate.

(5) (a) \( \lim_{x \to 0} \frac{6^x - 2^x}{x} = \lim_{x \to 0} \frac{(\ln 6)^x - (\ln 2)^2}{1} = \ln 6 - \ln 2 = \ln 3, \) by l’Hospital’s rule for form

\( 0/0. \)

(b) \( \lim_{x \to \pi} \frac{\sin(2x)}{1 - \cos(2x)} = \lim_{x \to \pi} \frac{\sin(2\pi)}{1 - 2 \cos(\pi)} = \lim_{x \to \pi} \frac{0}{3} = 0. \)

(c) \( \lim_{x \to 0} \frac{e^x - x - 1}{x^2} = \lim_{x \to 0} \frac{e^x - 1}{2x} = \lim_{x \to 0} \frac{e^x}{2} = \frac{1}{2}, \) by l’Hospital’s rule twice.

(6) Length \( L = 8x + 3y \) is to be minimized. The area \( (x + 3x) \cdot y = 9600, \) so \( y = 2400/x. \)

Therefore, \( L = 8x + 3(2400/x) = 8x + 7200/x \) for \( 0 < x < \infty. \) Now \( \frac{dL}{dx} = 8 - \frac{7200}{x^2}, \) and \( \frac{dL}{dx} = 0 \Leftrightarrow 8x^2 = 7200 \Rightarrow x^2 = 900 \Rightarrow x = 30, \) since \( x > 0. \) Here \( \frac{d^2L}{dx^2} = \frac{14400}{x^3} > 0, \) so \( L \) is minimized where \( \frac{dL}{dx} = 0, \) i.e., \( x = 30 \) feet and \( y = 80 \) feet.

(Alternative tests: (1) Check “endpoints”: \( L \to \infty \) as \( x \to 0^+ \) and as \( x \to \infty; \) (2) \( 0 < x < 30 \Rightarrow dL/dx = 8 - 7200/x^2 < 0 \) and \( x > 30 \Rightarrow dL/dx = 8 - 7200/x^2 > 0. \)

(7) (a) \( f'(x) = 12x^2 - 4x^3 = 4x^2(3-x), \) so the critical points are \( x = 0 \) and \( x = 3. \) If \( x < 0, f'(x) > 0; \) if \( 0 < x < 3, f'(x) > 0; \) if \( x > 3, f'(x) < 0. \) Therefore \( f \) is increasing on \((-\infty, 3), \) and \( f \) is decreasing on \([3, \infty). \)

(b) (a) also shows that \( f(3) = 27 \) is a local maximum, and there is no local minimum.

(c) \( f''(x) = 24 - 12x^2 = 12x(2-x), \) and \( f''(x) \) changes sign from \(-\to + \to - \) at \( x = 0 \) and \( x = 2, \) so these \( (0, 0) \) and \((2, 16)\) are the inflection points, and \( f \) is concave up on \([0, 2]\) and concave down on \((-\infty, 0]\) and on \([2, \infty). \)
(8) (a) Let \( q = \) the number of passengers; let \( p = \) the price. Then
\[
p = \begin{cases} 
2000 & \text{if } 0 \leq q \leq 100 \\
2000 - 10(q - 100) = 3000 - 10q & \text{if } 100 < q \leq 200,
\end{cases}
\]
so revenue
\[
R = pq = \begin{cases} 
2000q & \text{if } 0 \leq q \leq 100 \\
3000q - 10q^2 & \text{if } 100 < q \leq 200.
\end{cases}
\]
Therefore, marginal revenue
\[
\frac{dR}{dq} = \begin{cases} 
2000 & \text{if } 0 \leq q < 100 \\
\text{undefined} & \text{if } q = 100 \\
3000 - 20q & \text{if } 0 < q \leq 200,
\end{cases}
\]
and \( \frac{dR}{dq} = 0 \) if and only if \( 3000 - 20q = 0 \), i.e., \( q = 150 \). Checking the endpoints (0 and 200) and critical points (100 and 150) we find

<table>
<thead>
<tr>
<th>( q )</th>
<th>0</th>
<th>100</th>
<th>150</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R )</td>
<td>$0$</td>
<td>$200,000$</td>
<td>$350,000$</td>
<td>$200,000$</td>
</tr>
</tbody>
</table>

Thus, revenue is maximized when Trump has 150 passengers, and the price is then $1500.

(b) Total cost \( C = 100,000 + 500q \) (0 \leq q \leq 200).