MCS121 Exam 2, Spring, 2005

This test is a closed-book test; you are allowed one 3" x 5" note card and a graphing calculator. Please write your name below. Look at all of the problems before deciding which to do first. Note that some problems are easier than others, and some are worth more points. In order to earn full credit, your solution steps must be clearly presented. Be sure to use notation correctly.

You may use the back of the pages if you need additional space, and you may request additional paper. You have one hour and fifty-five minutes to work.

Name: _____

Instructor: (circle one)

Holte at 9:00 Rietz at 11:30 Hvidsten at 10:30

Holte at 2:30 Rietz at 1:30

Problem	Page	Possible	Score
1	2	15	
2	2	12	
3	3	12	
4	3	10	
5	4	15	
6	4	12	
7	5	15	
8	5	9	
Total		100	

Honor pledge: Please consider signing the following honor pledge:

On my honor, I pledge that I have not given, received, or tolerated others' use of unauthorized aid in completing this work.

Name: _____

1. [**15 Points**] Let $y = f(x) = 4x^5 + 5x^4$.

   (a) Find the critical points of f and determine where f is increasing and decreasing.

   (b) Find all local maxima and minima for f.

   (c) Find the inflection point(s) for f and determine where f is concave up and concave down.

2. [**12 Points**] Consider the function $f(x) = e^{1+x^2}$. Find the x-values for the global maximum and minimum on the interval $[-1, 2]$.

3. [**12 Points**]

(a) What is the tangent line approximation (local linearization) to $f(x) = \dfrac{1}{\sqrt{x}}$ near $x = 4$?

(b) For x near 4 but $x \neq 4$, will this tangent-line approximation yield an overestimate or an underestimate to the value of $f(x)$? Explain your answer.

(c) Use the tangent-line approximation to estimate $f(5) = \dfrac{1}{\sqrt{5}}$.

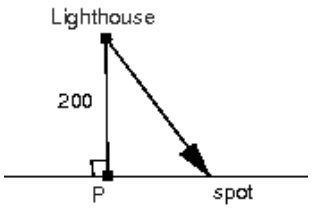
4. [**10 Points**] Evaluate the following limits exactly, if possible. If a limit does not exist, say so.

(a) $\lim\limits_{x \to 3} \dfrac{x^2 - 4x + 3}{x^2 + x - 12}$

(b) $\lim\limits_{x \to 0} \dfrac{1 + e^x}{e^{1+x}}$

(c) $\lim\limits_{x \to 0} \dfrac{\cos(x) - 1}{x^2}$

5. [**15 Points**] A lighthouse is located 200 meters away from a point P on a long, straight shoreline. It has a beacon that makes three revolutions per minute (6π radians per minute). As its beam of light sweeps along the shoreline, the spot illuminated on the shoreline moves faster and faster as it gets farther and farther from P. How fast is the illuminated spot moving when the angle between the beam of light and the line from the lighthouse to P is $\frac{\pi}{4}$ radians?



6. [**12 Points**] The sum of two positive numbers x and y is 10. How should these numbers be chosen so that the number $x + \ln(y)$ is maximized? What is the maximum value? Show that your answer is a maximum.

7. [**15 Points**] A rectangular wooden box, with a square base and a metal lid, is to have volume 288 cubic inches. The wood for the sides and the base costs 3 cents per square inch and the metal for the lid costs 5 cents per square inch. What dimensions of the box will minimize the total cost of the materials needed to build the box? Show that your answer gives a minimum cost.

8. [**9 Points**] The revenue from selling q items is $R(q) = 625q - \frac{q^3}{3}$. The total cost for producing q items is $C(q) = 127 + 96q$.

   (a) What is the fixed cost of production here?

   (b) What is the profit function $\pi(q)$?

   (c) Find the quantity that maximizes the profit. Show that your answer does give the maximum profit.