MCS121
Answers to homework: 1.3, 1.4

1.3.2 (a) \( f(t + 1) = (t + 1)^2 + 1 = t^2 + 2t + 2 \)
(b) \( f(t^2 + 1) = (t^2 + 1)^2 + 1 = t^4 + 2t^2 + 2 \)
(c) \( f(2) = 5 \)
(d) \( 2f(t) = 2(t^2 + 1) = 2t^2 + 2 \)
(e) \( (f(t))^2 + 1 = (t^2 + 1)^2 + 1 = t^4 + 2t^2 + 2 \)

1.3.4 (a) \( y = 3 \)

1.3.6 \( m(z + h) - m(z) = (z + h)^2 - z^2 = 2zh + h^2 \)

1.3.10 (a) \( f(10,000) \) represents the value of \( C \) corresponding to \( A = 10,000 \), or in other words the cost of building a 10,000 square-foot store.
(b) \( f^{-1}(20,000) \) represents the value of \( A \) corresponding to \( C = 20,000 \), or in other words the area in square feet of a store which would cost $20,000 to build.

1.3.14 The function is not invertible. It fails the horizontal line test.

1.3.26 \( g(f(2)) \approx g(0.4) \approx 1.1. \)

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<th>( x )</th>
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<th>( g(x) )</th>
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1.4.8 \( x = \frac{\log(2/11)}{\log(7/5)} \approx -5.07 \)
\[ \ln(10^{x+3}) = \ln(5e^{7-x}) \]
\[ (x+3)\ln 10 = \ln 5 + (7 - x)\ln e \]
\[ \ln 10x + 3\ln 10 = \ln 5 + 7 - x \]
\[ (\ln 10 + 1)x = -3\ln 10 + \ln 5 + 7 \]
\[ x = \frac{(-3\ln 10 + \ln 5 + 7)/(\ln 10 + 1)}{0.515} \]
also \[ x = \frac{(\log 5 + 7\log e - 3)/(1 + \log e)}{0.515} \]

1.4.36 We know that the \( y \)-intercept of the line is at \((0,1)\), so we need one other point to determine the equation of the line. We observe that it intersects the graph of \( f(x) = 10^x \) at the point \( x = \log 2 \). The \( y \)-coordinate of this point is then \( y = 10^x = 10^{\log 2} = 2 \), so \((\log 2, 2)\) is the point of intersection. We can now find the slope and equation of the line. \( y = \frac{1}{\log 2}x + 1 \approx 3.3219x + 1. \)

1.4.38 (a) Since the initial amount of caffeine is 100mg and the exponential decay rate is \(-0.17\), we have \( A = 100e^{-0.17t} \).

(b) We estimate the half-life by estimating \( t \) when the caffeine is reduced by half (so \( A = 50 \)); this occurs at approximately \( t = 4 \) hours.

(c) We want to find the value of \( t \) when \( A = 50 \);
\[
50 = 100e^{-0.17t} \\
0.5 = e^{-0.17t} \\
\ln 0.5 = -0.17t \\
t = \frac{\ln 0.5}{-0.17} \approx 4.077.
\]

1.4.46 Let \( m \) be the infant mortality of Senegal. As a function of time \( t \) (in years), \( m \) is given by \( m = m_0(0.90)^t \). To find when \( m = 0.5m_0 \) (so the number of cases has been reduced by 50%), we solve
\[
0.5m_0 = m_0(0.90)^t \\
0.5 = (0.90)^t \\
\ln(0.5) = t \ln(0.9) \\
t = \frac{\ln(0.5)}{\ln(0.9)} \approx 6.58 \text{ years}.
\]