Solution for Global Minimax worksheet

1. end points : x = 0, 50 critical points : 50

$$\begin{aligned} A(0) &= 0 \quad \longleftarrow \quad \text{global min} \\ A(50) &= 2500 \quad \longleftarrow \quad \text{global max} \end{aligned}$$

2. end points : $t = 0, 3$ critical points : $t = 1$

$$\begin{aligned} h(0) &= 1 \\ h(1) &= -1 \quad \longleftarrow \quad \text{global min} \\ h(3) &= 19 \quad \longleftarrow \quad \text{global max} \end{aligned}$$

3. end points : $q = -10, 100$ critical points : $q = 50$

$$\begin{aligned} R(-10) &= 24 \quad \longleftarrow \quad \text{global max} \\ R(50) &= -12 \quad \longleftarrow \quad \text{global min} \\ R(100) &= 13 \end{aligned}$$

4. end points : $x = 0, 1$ critical points : $x = \frac{1}{2}$

$$\begin{aligned} D(0) &= 1 \quad \longleftarrow \quad \text{global max} \\ D\left(\frac{1}{2}\right) &= \frac{\sqrt{3}}{2} \approx 0.866 \quad \longleftarrow \quad \text{global min} \\ D(1) &= 1 \quad \longleftarrow \quad \text{global max} \end{aligned}$$

5. end points : $x = 0, \pi$ critical points : $x = \frac{\pi}{4}$

$$\begin{aligned} g(0) &= 1 \\ g\left(\frac{\pi}{4}\right) &= \sqrt{2} \approx 1.414 \quad \longleftarrow \quad \text{global max} \\ g(\pi) &= -1 \quad \longleftarrow \quad \text{global min} \end{aligned}$$

6. end points : none critical points : $r = \sqrt[3]{\frac{20}{4\pi}}$

$$\lim_{r \to 0^+} s(r) = \infty \qquad \lim_{r \to \infty} s(r) = \infty$$

Using the second derivative test, $s''\left(\sqrt[3]{\frac{20}{4\pi}}\right) = 12\pi > 0$, so $r = \sqrt[3]{\frac{20}{4\pi}}$ is a local min and hence a global min. $s(r)$ has no global max.

7. end points : $x = 1, 8$, critical points : none.

$$\begin{aligned} f(1) &= 1 \quad \longleftarrow \quad \text{global min} \\ f(8) &= 2 \quad \longleftarrow \quad \text{global max} \end{aligned}$$

8. end points : none critical points : $x = \dfrac{8\sqrt{27}}{\sqrt{8}+\sqrt{27}} = x_0$

$$\lim_{x\to 0^+} p(x) = \infty \qquad \lim_{x\to 8^-} p(x) = \infty.$$

Using the second derivative test, $p''(x_0) > 0$, so x_0 is a local min and hence a global min; $p(x)$ has no global max.