Please indicate the brand and model number of your calculator on the line above. Besides a calculator, you may use one handwritten page of notes (to be turned in with your test) and the page, “Handy Identities for Integration.” Use notation correctly, including “=,” “≈,” “→,” and “⇒”; write equalities where appropriate, but not where they would be incorrect. Indicate clearly how you get your answers.

There are twelve problems, each worth 20 points. Do ten (10) of the twelve problems. Indicate below which ten problems you want graded.

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1. Calculate the following integrals.

\[
\int \frac{1}{2x} \, dx \\
\int \frac{1}{x^2} \, dx \\
\int \sin^2 x \, dx \\
\int \frac{1}{\sin^2 x} \, dx \\
\int \frac{1}{\sqrt{1 - 4x^2}} \, dx
\]

2. Sketch the region bounded by the graph of \( y = (\pi/2)^2 - x^2 \) and the graph of \( y = \cos x \), and find its area.

3. Find the volume of the solid formed by revolving the region bounded by the graphs of \( y = 2x \) and \( y = x^2 \) about the \( x \)-axis.

4. \( \int x^2 e^{3x} \, dx = ? \)

5. \( \int_1^2 \frac{1}{x^2 \sqrt{9 - x^2}} \, dx = ? \)

6. Describe the procedure you would follow to integrate an arbitrary rational function whose denominator has only linear (first-degree) factors, such as

\[
\int \frac{x^6 + 5x^4 - 7}{(x - 2)(x - 1)^3} \, dx \quad \text{or} \quad \int \frac{x^2 + 6}{(2x + 5)^2(3x - 1)(x + 4)^2} \, dx.
\]

(Do not actually integrate these examples.) Include in your description the two or three integration formulas that each such problem can be reduced to.
7. Evaluate the following expressions to within 0.01, or one cent.

a. The present value of an annuity that will pay $500 per year for twenty years starting one year from now, assuming an annual interest rate of 6%.
(Note: the present value of a payment of $R$ to be paid $n$ years from now is $\frac{R}{(1 + i)^n}$ if the annual interest rate is $i$.)

b. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$

8. Determine the set of $x$ values for which the following power series converges:

$$\sum_{n=2}^{\infty} \frac{(x - 3)^n}{2^n \ln n}.$$ 

Show your supporting work, and cite by name the tests you use.

9. 

a. Determine the Maclaurin series for $\cos(x^2)$.

b. Evaluate

$$\int_{0}^{1} \cos(x^2) \, dx$$

numerically with an error less than 0.0005. Justify your answer.

10. A point moves in the plane in such a way that its position $(x, y)$ at time $t$ is given by

$$x = t^3 \quad \text{and} \quad y = 3t^2/2.$$ 

a. Its velocity is defined to be $(\frac{dx}{dt}, \frac{dy}{dt})$. Compute the velocity of the point at time $t = 1$.

b. Find an equation for the tangent line to the curve traced by the point at the point when $t = 1$.

c. How far does the point travel between times $t = 0$ and $t = \sqrt{3}$?
11. Carefully draw the graph of the polar equation

\[ r = \frac{6}{2 + 4\cos \theta} \]

on the polar coordinate paper provided. Show your table of values and other supporting work on this page. In addition, determine an equivalent Cartesian (\(x\) and \(y\)) equation for the graph, and identify what type of curve it is.

12. Find the area enclosed by the graph of \(r = 2\sin(3\theta)\) in polar coordinates.

Have a great summer!!