STATISTICAL INFERENCE FOR ONE-WAY ANALYSIS OF VARIANCE

0. Assume that the data

Sample from population 1	Sample from population 2	...	Sample from population I
x_{11}	x_{21}	...	x_{I1}
x_{12}	x_{22}	...	x_{I2}
x_{13}	x_{23}	...	x_{I3}
...	...	...	...
x_{1n_1}	x_{2n_2}	...	x_{In_I}

are values of independent normal random variables X_{ij} ($i = 1, \ldots, I$ and $j = 1, \ldots, n_i$) with mean μ_i and constant standard deviation σ. Alternatively, each $X_{ij} = \mu_i + \epsilon_{ij}$ where the ϵ_{ij}'s are independent $N(0, \sigma)$ random errors. Let $N = n_1 + n_2 + \ldots + n_I$, the total number of observations.
The parameters of this model are the population means $\mu_1, \mu_2, \ldots, \mu_I$ and the common standard deviation σ.

1. Test $H_0 : \mu_1 = \mu_2 = \ldots = \mu_I$ against H_a : not all of the means μ_i are equal.

2. The test statistic is the F statistic (named for statistical pioneer Sir Ronald Fisher). This is traditionally displayed in an ANOVA (ANalysis Of Variance) table, even though we need only the F value and its two degrees of freedom.

Source of variation	Degrees of freedom	Sum of squares	Mean square	F value	P
Between groups (model)	$DFG = I - 1$	$SSG = \sum n_i(\overline{x}_i - \overline{x})^2$	$MSG = SSG/DFG$	$F = MSG/MSE$	
Within groups (error)	$DFE = N - I$	$SSE = \sum (n_i - 1)s_i^2$	$MSE = SSE/DFE$		
Total	$DFT = N - 1$	$SST = \sum_{i,j} (x_{ij} - \overline{x})^2$	$MST = SST/DFT$		

3. Under the model assumptions and the null hypothesis, the F statistic has an F distribution. Critical values of the F statistic are given in Table E of our text. There are two parameters: the numerator degrees of freedom and the denominator degrees of freedom. To do one-way ANOVA on the TI-83 Plus, enter the data into lists and then do this: STAT TESTS F:ANOVA($L_1, L_2, \ldots, L_I$) ENTER. This will give you a P value, too.

This procedure is called "analysis of variance" because, it turns out, SST = SSG + SSE, i.e., the total sum of squares equals (is "analyzed" into) the between-groups sum of squares and the within-groups (error) sum of squares.