

## STATISTICAL INFERENCE FOR SIMPLE LINEAR REGRESSION

Assume that the data  $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$  are observations either of random points  $(X, Y)$  or points  $(x, Y)$  with random ordinates and that  $Y_1, Y_2, \dots, Y_n$  are independent, normally distributed random variables with mean

$$E(Y_i|x_i) = \alpha + \beta x_i = \beta_0 + \beta_1 x_i$$

for each given  $x_i$  and *constant* standard deviation  $\sigma$ . Alternatively,  $Y_i = \beta_0 + \beta_1 x_i + \epsilon_i$  where the  $\epsilon_i$ 's are independent normal random variables (errors) with mean 0 and standard deviation  $\sigma$ . In brief, the  $Y_i$ 's are independent and  $N(\alpha + \beta x_i, \sigma)$ .

**General pattern for inference:** To test  $H_0 : \theta = \theta_0$ , use the statistic  $T = \frac{\hat{\theta} - \theta_0}{SE_{\hat{\theta}}}$ .

The endpoints of a confidence interval for  $\theta$  are given by  $\hat{\theta} \pm t^* SE_{\hat{\theta}}$  where  $df = n - 2$ . Here  $SE_{\hat{\theta}}$  denotes the standard error of the statistic  $\hat{\theta}$ .

Parameter	Estimate/statistic	Standard Error
$\rho$	$r = \frac{1}{n-1} \sum \left( \frac{x_i - \bar{x}}{s_x} \right) \left( \frac{y_i - \bar{y}}{s_y} \right)$	
$\beta = \beta_1$	$\hat{\beta}_1 = r \frac{s_y}{s_x} = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(x_i - \bar{x})^2}$	$SE_{\hat{\beta}_1} = \frac{s}{\sqrt{\sum(x_i - \bar{x})^2}} = \frac{s}{s_x \sqrt{n-1}}$
$\alpha = \beta_0$	$\hat{\beta}_0 = \bar{y} - b\bar{x}$	$SE_{\hat{\beta}_0} = s \sqrt{\frac{1}{n} + \frac{\bar{x}^2}{\sum(x_i - \bar{x})^2}}$
$\mu_{y x} = E(Y x)$	$\hat{\mu} = \hat{\mu}_{y x} = \hat{\beta}_0 + \hat{\beta}_1 x$	$SE_{\hat{\mu}} = s \sqrt{\frac{1}{n} + \frac{(x - \bar{x})^2}{\sum(x_i - \bar{x})^2}}$
$y$ (not actually a parameter)	$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$ (predicted value)	$SE_{\hat{y}} = s \sqrt{1 + \frac{1}{n} + \frac{(x - \bar{x})^2}{\sum(x_i - \bar{x})^2}}$ (standard error for <i>predicted</i> value)
$\sigma$	$s = \sqrt{\frac{\sum(y_i - \hat{y}_i)^2}{n-2}}$	