MCS 142: INTRODUCTION TO STATISTICS
SECTION 4.3 and 4.4 HOMEWORK

(1) Let $X$ be a random variable representing the outcome when a single die is rolled once.
   (a) Find a formula for the probability distribution of the random variable $X$.
   (b) Find the mean, $\mu$, of the random variable $X$ and interpret it in context.
   (c) Find the variance of $X$ using $\sigma^2 = E(X - \mu)^2$.
   (d) Find the variance of $X$ using $\sigma^2 = E(X^2) - \mu^2$.

(2) The proportion of people who respond to a certain mail-order solicitation is a continuous random variable $X$ with pdf (probability density function)
   
   \[ f(x) = \begin{cases} 
   \frac{2(x+2)}{5} & 0 < x < 1 \\
   0, & \text{otherwise} 
   \end{cases} \]

   (a) Show that $P(0 < X < 1) = 1$.
   (b) What is the probability that more than $1/4$ but fewer than $1/2$ of the people contacted will respond to this type of solicitation?

(3) Let $X$ denote the maximum of three real numbers chosen independently and uniformly (i.e., without bias) at random from the interval $[1, 10]$. The pdf of $X$ is $f$ where

   \[ f(x) = \frac{(x - 1)^2}{243} \quad \text{if} \quad 1 \leq x \leq 10. \]

   (a) What is the probability that $X$ is less than 5?
   (b) What is the probability that $X$ exceeds 5?
   (c) What is the probability that $X = 5$?
   (d) What is the probability that $4 < X \leq 6$?
   (e) What is the mean value of $X$?
   (f) What is the variance of $X$?
   (g) What is the standard deviation of $X$?

(4) One commonly used model of component lifetimes is the exponential model. Suppose that the lifetime $L$ of a component, in hours, is a random variable having pdf

   \[ f(x) = 0.01e^{-0.01x} \quad \text{if} \quad x > 0. \]

   Find the probability that the component’s lifetime is between 100 and 200 hours.
CALCULUS REVIEW

THE FUNDAMENTAL THEOREM OF CALCULUS

If \( f \) is a continuous function on \([a, b]\) and if \( F \) is any antiderivative of \( f \), then
\[
\int_a^b f(x) \, dx = F(b) - F(a).
\]

A SHORT TABLE OF INTEGRALS

The indefinite integral \( \int f(x) \, dx \) represents an arbitrary antiderivative of \( f \). Let \( C \) denote
an arbitrary constant. Over any interval on which the integrand is defined we have:
\[
\int x^n \, dx = \frac{x^{n+1}}{n+1} + C \quad \text{if} \quad n \neq -1,
\]
\[
\int \frac{1}{x} \, dx = \ln |x| + C,
\]
\[
\int e^x \, dx = e^x + C \quad \text{if} \quad b \neq 0
\]
and
\[
\int b^x \, dx = \frac{b^x}{\ln b} + C \quad \text{if} \quad b > 0, b \neq 1.
\]

CHANGE OF VARIABLE/SUBSTITUTION

Let \( u = g(x) \). Then \( du = g'(x) \, dx \) and
\[
\int f(g(x)) \cdot g'(x) \, dx = \int f(u) \, du
\]
and
\[
\int_a^b f(g(x)) \cdot g'(x) \, dx = \int_{g(a)}^{g(b)} f(u) \, du.
\]