Taylor Series

Let $f$ be a function that has derivatives of every order at the number $a$. Then the Taylor series of \( f(x) \) about $x = a$ is the power series

$$
\sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!} (x - a)^k.
$$

When $a = 0$ the Taylor series is sometimes also called a Maclaurin series, and it is given by

$$
\sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{k!} x^k.
$$

For so-called analytic functions (which are the ones we most commonly meet),

$$
f(z) = \sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{k!} z^k
$$

for $z$ in the interval of convergence of the power series. Hence, we have

$$
[z^k] f(z) = \frac{f^{(k)}(0)}{k!}.
$$