Counting Notation

1. The number of elements in set \( A \) is denoted \( \#(A) \) or \( |A| \).

2. For integer \( k \),
   \[
   \binom{r}{k} = \begin{cases} \frac{r^k}{k!} & \text{if } k \geq 0 \\ 0 & \text{if } k < 0. \end{cases}
   \]
   For a positive integer \( r \), “\( r \) choose \( k \)” counts the number of \( k \)-element subsets of an \( r \)-element set.

3. A multiset is just like a set except that repetitions count (make a difference). One notation for a multiset lists in braces each element preceded by its repetition factor and a centered dot. For example,
   \[
   \{S, T, A, T, I, S, T, I, C, S\} = \{1 \cdot A, 1 \cdot C, 2 \cdot I, 3 \cdot S, 3 \cdot T\}.
   \]

4. The number of (unordered) selections of \( k \) items from a multiset of \( n \) distinct elements with no restriction on the number of repetitions (any number from 0 to \( k \) of each item may be taken), read “\( n \) multichoose \( k \)”, is
   \[
   \binom{n}{k} = \binom{k + n - 1}{k}.
   \]

5. The Stirling number of the second kind, \( \begin{bmatrix} n \\ k \end{bmatrix} \), read “\( n \) subset \( k \)”, gives the number of ways to partition a set of \( n \) elements into \( k \) (unordered) nonempty subsets.
   For a given \( n \), the Stirling numbers of the second kind are just the right coefficients to use to express an ordinary \( n^{th} \) power in terms of falling powers:
   \[
   x^n = \sum_k \begin{bmatrix} n \\ k \end{bmatrix} x^k.
   \]

6. For integer \( k \geq 2 \) and nonnegative integers \( r_1, \ldots, r_k \) satisfying \( r_1 + \cdots + r_k = n \), the multinomial coefficient is denoted and defined as follows:
   \[
   \binom{n}{r_1, \ldots, r_k} = \frac{n!}{r_1! \cdots r_k!}.
   \]

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