MCS 256 DISCRETE MATHEMATICS
Section 2.4 plus

Problems and Answers

1. Evaluate the following sums in closed form. (Allow $H_n$ in “closed-form” solutions.)

(a)\[
\sum_{k=1}^{n} \frac{2k - 1}{k(k + 1)} = 2H_n - 3 + \frac{3}{n + 1}.
\]

(b)\[
\sum_{0 \leq k < n} \frac{1}{(k + 2)(k + 3)(k + 4)} = (k + 1)\left[2^n - 1\right]_0 = \frac{n(n + 5)}{12(n + 2)(n + 3)}.
\]

2. Evaluate the following multiple sums in closed form.

(a)\[
\sum_{1 \leq i < j \leq n} 3^{i-j} = \frac{1}{2}n - \frac{3}{4} + \frac{3}{4}3^{-n}.
\]

(b)\[
\sum_{1 \leq i \leq n} 1 = n.
\]

\[
\sum_{1 \leq i < j \leq n} 1 = \frac{n(n - 1)}{2}.
\]

\[
\sum_{1 \leq i < j < k \leq n} 1 = \frac{n(n - 1)(n - 2)}{6}.
\]

(c)\[
\sum_{j=1}^{n} \sum_{k=j}^{n} \frac{1}{k^2} = H_n.
\]