1. (a) Use
\[ i^n = \begin{cases} 
1 & \text{if } n \equiv 0 \pmod{4} \\
i & \text{if } n \equiv 1 \pmod{4} \\
-1 & \text{if } n \equiv 2 \pmod{4} \\
-i & \text{if } n \equiv 3 \pmod{4} 
\end{cases} \]
to rewrite our solution of the recurrence
\[ g_0 = 0; \]
\[ g_1 = 1; \]
\[ g_2 = 2; \]
\[ g_3 = 3; \]
\[ g_n = -2g_{n-2} - g_{n-4} \quad \text{for } n > 3 \]
without any \( i \)'s.
(b) Evaluate \( g_{100}, g_{101}, g_{102}, \) and \( g_{103}. \)

2. Consider the recurrence
\[ g_n = -2g_{n-1} + 3g_{n-2} + 2^n. \]
(a) Find the general solution.
(b) Find the solution that satisfies the initial conditions \( g_0 = 2 \) and \( g_1 = 3. \)

3. Consider the recurrence
\[ g_n = -2g_{n-1} + 3g_{n-2} + 4. \]
(a) Find the general solution.
(b) Find the solution that satisfies the initial conditions \( g_0 = 3 \) and \( g_1 = 18. \)

4. Finish the proof of the linearity of the operator \( L \) defined on sequences \( \langle g_n \rangle \) by
\[ L(g_n) = g_n - \sum_{k=1}^{r} c_k g_{n-k}. \]

5. The number of comparisons \( C_n \) used by the Mergesort algorithm to sort a list of \( n \) items satisfies the recurrence
\[ C_n = C_{[n/2]} + C_{[n/2]} + n \quad \text{for } n \geq 2 \]
with \( C_1 = 0. \) This is a kind of recurrence that arises in “divide-and-conquer” algorithms. It is not of the form of the linear recurrence relations with constant coefficient that we have considered heretofore, because the difference between the largest and smallest indices, \( n - [n/2], \) is not a constant.
(a) Solve this recurrence in the special case that \( n \) is a power of 2, say \( n = 2^k, \) by finding a recurrence relation for \( b_k := \frac{1}{2} C_{2^k} \) and solving it.
(b) Show that $C_n = \Theta(n \log n)$ in the preceding case.
(c) Again allowing $n$ to be any positive integer, let $D_n = \Delta C_n = C_{n+1} - C_n$. Determine the simple recurrence that $D_n$ satisfies, solve it, and hence find $C_n$.
(d) (Extra credit) Find a formula for $C_n$ that does not involve the sum of a variable number of terms.