MCS 256 CONCRETE MATHEMATICS    Set 3

1. The purpose of this problem is to show you parallels and contrasts between differentiating and differencing. Evaluate the following derivatives and differences. In these problems, the derivative operator is denoted by \( D \), and the forward difference operator is denoted by \( \Delta \). Compile your answers in a table on one page, and attach your neatly written supporting work.

\[
\begin{align*}
Df(x) & \quad \Delta f(x) \\
D \! e^x & \quad \Delta e^x \\
D \! 2^x & \quad \Delta 2^x \\
D \! x^3 & \quad \Delta x^3 \\
D \! x^2 & \quad \Delta x^2 \\
D \! \ln x & \quad \Delta H_x \\
D \! \frac{1}{x} & \quad \Delta \frac{1}{x + 1}
\end{align*}
\]

2. The purpose of this problem is to show you parallels and contrasts between integration and summation. Evaluate the following definite integrals and sums. Assume that \( a \) and \( b \) are integers and \( n \) is a positive integer. Compile your answers in a table on one page, and attach your neatly written supporting work.

\[
\begin{align*}
\int_a^b Df(x) \, dx & \quad \sum_{a \leq k < b} \Delta f(k) \\
\int_a^b e^x \, dx & \quad \sum_{a \leq k < b} 2^k \\
\int_a^b 2^x \, dx & \quad \sum_{a \leq k < b} e^k \\
\int_0^n x^4 \, dx & \quad \sum_{0 \leq k < n} k^4 \\
\int_0^n x^4 \, dx & \quad \sum_{0 \leq k < n} k^4 \\
\int_0^n x^2 e^x \, dx & \quad \sum_{0 \leq k < n} k^2 2^k
\end{align*}
\]

Homework rules

- Acknowledge your sources (people and texts).
- In nontrivial problems, show how you get your answers.
- Turn in neat, well-written solutions, not messy first drafts. Trim "fringes."
- Do not copy collaborative solutions; write up solutions in your own words.
- Turn in homework on time. Each class day late reduces the possible points by 25%.
- Do extra credit problems on your own, without consulting any other person.
Extra credit problem

For the Josephus problem with people numbered 1, . . . , $n$ and every \textit{second} person eliminated in a circular fashion ($q = 2$), let $P(n)$ denote the number of the penultimate survivor. Find a closed-form formula for $P(n)$ and prove that it is correct.