MCS 256 DISCRETE MATHEMATICS  Set 5

Problems

1. Criticize these “solutions.”

\[ 4x \equiv 16 \pmod{26} \quad \implies 4x \equiv 15 \pmod{26} \]
\[ \implies 2x \equiv 8 \pmod{13} \quad \implies 8x \equiv 30 \equiv 4 \pmod{26} \]
\[ \implies 14x \equiv 56 \pmod{13} \quad \implies 4x \equiv 2 \pmod{13} \]
\[ \implies x \equiv 56 \pmod{13} \quad \implies 12x \equiv 6 \pmod{13} \]
\[ \implies x \equiv 4 \pmod{13} \quad \implies -x \equiv 6 \equiv -7 \pmod{13} \]
\[ \implies x \equiv 4 \quad \text{or} \quad x \equiv 17 \pmod{26} \quad \implies x \equiv 7 \quad \text{or} \quad x \equiv 20 \pmod{26} \]

(a) Check that there is no logical error. (See pp. 124–125.) Report your conclusion.
(b) According to the result stated in class, how many solutions does each congruence have?
(c) Which “solution” does not give a correct solution?
(d) Explain what is going on. Note at what point(s) the “funny business” occurs.

2. Calculate \( 12x \mod 21 \) for \( x = 0, 1, \ldots, 20 \). What regular pattern appears? Which proposition presented in class is illustrated?

3. Calculate \( 19^x \mod 21 \) for \( x = 0, 1, \ldots, 20 \). What regular pattern appears? Which proposition presented in class is illustrated?

4. Compute a table of values of the Euler totient function, \( \varphi(m) \), for \( m = 1, \ldots, 20 \).

5. (a) Find \( \gcd(57, 105) \) and integers \( m' \) and \( n' \) such that \( m' \cdot 57 + n' \cdot 105 = \gcd(57, 105) \).
(b) Find \( \gcd(64, 105) \) and integers \( m' \) and \( n' \) such that \( m' \cdot 64 + n' \cdot 105 = \gcd(64, 105) \).
(c) Solve \( 57x \equiv 87 \pmod{105} \).
(d) Solve \( 64x \equiv 83 \pmod{105} \).

Homework rules

- Acknowledge your sources (people and texts).
- In nontrivial problems, show how you get your answers.
- Turn in neat, well-written solutions, not messy first drafts. Trim ”fringes.”
- Do not copy collaborative solutions; write up solutions in your own words.
- Turn in homework on time. Each class day late reduces the possible points by 25%.
- You must do extra credit problems entirely on your own, without consulting with anyone.