MCS 256 DISCRETE MATHEMATICS
Set 9: Generating Functions

“We come now to the most important idea in this whole book, the notion of a generating
function.”  (196)

1. Complete the table of generating functions given out in class and available at
<http://www.gac.edu/~holte/courses/mcs256/2001S/documents/ogfs.ps> or

2. Let \( S_n = \sum_{k=0}^{n} c^k \) for integer \( n \geq 0 \). Determine the generating function of \( \langle S_n \rangle \).

3. The Lucas sequence \( \langle L_n \rangle \), named for the famous nineteenth century French mathematician Edouard Lucas, has the same recurrence as the Fibonacci sequence, but different
initial conditions:

\[
L_0 = 2; \\
L_1 = 1; \\
L_n = L_{n-1} + L_{n-2} \quad \text{for } n > 1.
\]

(a) Make a table showing the Fibonacci numbers \( F_n \) and Lucas numbers \( L_n \) for \( n = 0, 1, \ldots, 10 \).

(b) Find the generating function of \( \langle L_n \rangle \).

4. Consider the sequence \( \langle u_n \rangle \) given by:

\[
u_0 = 1; \\
u_1 = 2; \\
u_n = 3u_{n-1} + 4u_{n-2} \quad \text{for } n > 1.
\]

(a) Determine the generating function of this sequence.

(b) Find a closed-form formula for \( u_n \) valid for \( n \geq 0 \).

Homework rules

- Acknowledge your sources (people and texts).
- In nontrivial problems, show how you get your answers.
- Turn in neat, well-written solutions, not messy first drafts. Trim "fringes."
- Do not copy collaborative solutions; write up solutions in your own words.
- Turn in homework on time. Each class day late reduces the possible points by 25%.