

MCS 256 DISCRETE MATHEMATICS

COUNTING PROBLEMS

I.: Basic Principles

A.: Find a one-to-one correspondence between the set of all subsets of $\{a, b, c\}$ and the counting numbers $\{1, 2, \dots, \#\text{subsets}\}$.

B.: If an allowed symbol can be a letter (with upper-case and lower-case letters distinguished) or a digit, how many allowed symbols are there?

B.3./I.D./II.C.: How many five-letter strings formed from the alphabet $\{a, b, c\}$ have at least one of these letters missing?

B.4.: Pick a number k between 1 and 26; then pick a letter from the first k letters of the alphabet. How many different outcomes are possible?

C.: Pick a letter; then pick a digit. How many different outcomes are possible?

D.: How many different truth tables are possible for Boolean expressions involving n Boolean variables?

E.: What is the probability that a random permutation is a derangement?

F.: Let $C(n, k)$ denote the number of different ways to choose k items from a set of n items. Express $C(n, k)$ via ...

1: an explicit formula,

2: a recurrence relation,

3: a generating function.

II.: Linearly Ordered Arrangements

Assume a 26-letter alphabet is used.

A.: How many different five-letter strings consisting of five distinct letters are possible?

B.: How many different five-letter strings are possible if letters may be repeated?

C.1.: How many different permutations of the letters of the word “STATISTICS” are there?

C.2.: How many distinct 3-permutations of the multiset $\{3 \cdot A, 2 \cdot B, 1 \cdot C\}$ are there?

C.3.: How many different five-letter strings are possible if the string can have no more than two A 's, at most one B , and no more than three C 's?

III.: Unordered Selections

We are at a shop that offers 26 different flavors of ice cream.

A.1.: How many different five-scoop dishes are possible when all scoops have to be different flavors?

A.2.: How many different k -tuples (i_1, \dots, i_k) of integers are there that satisfy $1 \leq i_1 < \dots < i_k \leq n$?

B.1.: How many different five-scoop dishes are possible if there is no prohibition against repeating flavors?

B.2.: How many different nonnegative integer solutions $(x_1, x_2, \dots, x_{26})$ of the equation $x_1 + x_2 + \dots + x_{26} = 5$ are there?

C.: How many different five-scoop dishes are possible if there can be no more than two vanilla scoops, at most one chocolate scoop, and no more than three strawberry scoops?

IV.: Sampling

An urn contains balls numbered 1 through 9. We draw four at random. How many outcomes are possible if ...

- (1) ... we draw (and observe) the balls one at a time, replacing each ball after it is observed?

- (2) ... we draw (and observe) the balls one at a time, *not* replacing each ball after it is observed?

- (3) ... we draw the balls one at a time, replacing each ball after it is drawn, but do not keep track of the order in which the balls are drawn?

- (4) ... we draw all four balls at once without replacement?

V.: Distributions/Functions

A.: Distinguishable boxes

A.1.: How many ways can one give nine different gifts to three (different) people?

A.2: How many different ways can you put six indistinguishable articles into three distinct boxes?

B.: Distinguishable boxes, given numbers in each box

B.1: How many ways can one distribute nine different books to three kids, giving four to Al, three to Bea, and two to Cee?

B.1: (again) How many different deals are possible in bridge? In other words, how many ways can one deal 52 cards to four people, giving each person a 13-card hand?

B.?: When one puts six (indistinguishable) particles at random into states A, B, and C, what is the probability that there will be two particles in A, one in B, and three in C?

C.: Indistinguishable boxes

C.1.a: How many ways can you partition a set of eight distinct items into three nonempty subsets? Or how many ways can you divide into (i) three (nonempty) piles or (ii) at most three piles a Tootsie Pop, a Crunch bar, a package of m&m's, a lollipop, a Nutrageous bar, a Snickers bar, a Mr. Goodbar, and a Baby Ruth?

C.1.b: How many (unordered) partitions of a set of eight elements are possible?

C.2: Enumerate (list) all possible partitions of 7.

D.: How many cells in the Sixteen Cases can you figure out?