

The Principle of Mathematical Induction

“The most useful, and usually the simplest, general proof technique for our work is **mathematical induction**. Mathematical induction is also the principal proof technique in computer science.”

--Alan Tucker [*Applied Combinatorics*, Wiley, 1980, p. 7]

This principle provides a way to prove that a statement $S(n)$ about integer n is true for every $n \geq$ some particular integer n_0 . Four forms of the principle of mathematical induction are given below. The first two are sometimes called “strong induction.” In all these formulations, the variables k , n , and n_0 are integer-valued variables.

If $S(n)$ is true for $n = n_0$
and if for every $k > n_0$
the truth of $S(n)$ for $n_0 \leq n < k$
implies the truth of $S(n)$ for $n = k$,
then $S(n)$ is true for every $n \geq n_0$.

If $S(n)$ is true for $n = n_0$
and if for every $k \geq n_0$
the truth of $S(n)$ for $n_0 \leq n \leq k$
implies the truth of $S(k+1)$,
then $S(n)$ is true for every $n \geq n_0$.

If $S(n)$ is true for $n = n_0$
and if for every $k > n_0$
the truth of $S(k-1)$
implies the truth of $S(k)$,
then $S(n)$ is true for every $n \geq n_0$.

If $S(n)$ is true for $n = n_0$
and if for every $k \geq n_0$
the truth of $S(k)$
implies the truth of $S(k+1)$,
then $S(n)$ is true for every $n \geq n_0$.

Proof boilerplate

Check that $S(n_0)$ is true.

Let $k > n_0$.

Assume $S(n)$ for $n_0 \leq n < k$.

Deduce $S(k)$.

Conclude $S(n)$ for $n \geq n_0$.