Discrete Distributions and Summation Formulas

Corresponding to each family of discrete distributions there is a useful summation formula. The summation formula implies that the sum of the probabilities in each case is exactly 1. (Alternatively, assuming that the probability function is indeed a *probability* function, the formula

$$\sum_x P(X = x) = 1$$

can be used to deduce the summation formula for valid values of the parameters. But notice that the summation formulas may hold even for other values of the parameters.) **These summation formulas are useful in calculating the mean, variance, and other moments of each distribution.**

Probability distribution & parameters	Probability function $P(X = x)$	Summation formula
Binomial $n = 1, 2, 3, \ldots$ $0 < p < 1$	$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$ $(x = 0, 1, 2, \ldots, n)$	Binomial theorem $\sum_{k=0}^{n} \binom{n}{k} p^k q^{n-k} = (p+q)^n$ (real p, q, integer $n \geq 0$)
Geometric $0 < p < 1$	$P(X = x) = p(1-p)^x$ $(x = 0, 1, 2, \ldots)$	Geometric series sum $\sum_{k=0}^{\infty} ar^k = \frac{a}{1-r}$ (real a, $\lvert r \rvert < 1$)
Negative binomial $r = 1, 2, 3, \ldots$ $0 < p < 1, q = 1 - p$	$P(X = x) = \binom{-r}{x} p^r (-q)^x$ $(x = 0, 1, 2, \ldots)$	(Newton's) binomial series $\sum_{k=0}^{\infty} \binom{\alpha}{k} t^k = (1+t)^\alpha$ (real α, $\lvert t \rvert < 1$)
Hypergeometric $N = 1, 2, 3, \ldots$ $a = 0, 1, 2, \ldots, N$ $n = 0, 1, 2, \ldots, N$	$P(X = x) = \binom{a}{x}\binom{N-a}{n-x} \Big/ \binom{N}{n}$ $(x = 0, 1, 2, \ldots n \wedge a \wedge (N-a))$	Vandermonde identity $\sum_k \binom{a}{k}\binom{b}{n-k} = \binom{a+b}{n}$ (real a, b, integer n)
Poisson $\lambda > 0$	$P(X = x) = e^{-\lambda}\frac{\lambda^x}{x!}$ $(x = 0, 1, 2, \ldots)$	Exponential series $\sum_{k=0}^{\infty} \frac{t^k}{k!} = e^t$ (real t)