Exam 2 Answers

(1) $H_n \sim \ln n \asymp \lg n \prec n^e \prec e^n \prec (n/e)^n \prec \sqrt{2\pi n}(n/e)^n \sim n!$.

(2) (a) $\lim_{x\to\infty} \ln x = \infty$ and $\lim_{x\to\infty} x = \infty$. By L'Hospital's rule, $\lim_{x\to\infty} \frac{\ln x}{x} = \lim_{x\to\infty} \frac{1/x}{1} = 0$.

(b) $\ln n^{1/n} = \frac{\ln n}{n} \to 0$, by (a), so $n^{1/n} \to e^0 = 1$, by continuity.

(c) Note: $n^{-1/n} \to 1$. So: T, T, F.

(3) $26 + 26 = 52$. $52 + 10 = 62$. $52(62^5 + 62^6 + 62^7) \approx 1.861 \times 10^{14}$.

(4) $(26 + 26^2 + 26^3)(10 + 10^2 + 10^3) = 20,288,580$.

(5) (a) $\binom{52}{5} = 2,598,960$.

(b) $13 \cdot \binom{4}{3} \cdot 12 \cdot \binom{4}{2} / \binom{52}{5} = 3744/2598960 \approx 1.441 \times 10^{-3}$.

(6) $2 + 2^2 + 2^3 + 2^4 = 30$.

(7) It's the same as the number of nonnegative solutions of $x_1 + x_2 + x_3 + x_4 = 6$, i.e., $\left(\binom{4}{6}\right) = \binom{9}{3} = 84$. (3 bars and 6 stars)

(8) (a) $3^6 = 729$.

(b) $\frac{6!}{1!2!3!} = 60$.

(c) $\left\{ {6 \atop 3} \right\} = 90$.

n k	1	2	3
1	1		
2	1	1	
3	1	3	1
4	1	7	6
5	1	15	25
6	1	31	90

$$\left\{ {n \atop k} \right\} = \left\{ {n-1 \atop k-1} \right\} + k \left\{ {n-1 \atop k} \right\}.$$