MCS 256 DISCRETE CALCULUS

Section 2.4 Plus: Answers

Problems and Answers

(1) Evaluate the following sums in closed form. (Allow $H_n$ in “closed-form” solutions.)

(a) \[
\sum_{k=1}^{n} \frac{2k - 1}{k(k + 1)} = 2H_{n+1} - 3 + \frac{1}{n+1}.
\]

(b) \[
\sum_{0 \leq k < n} \frac{1}{(k + 2)(k + 3)(k + 4)} = \frac{(k + 1)^2}{-2} \bigg|_{0}^{n} = \frac{n(n + 5)}{12(n + 2)(n + 3)}.
\]

(2) Evaluate the following multiple sums in closed form.

(a) \[
\sum_{1 \leq i < j \leq n} 3^{i-j} = \frac{1}{2} n - \frac{3}{4} + \frac{3}{4} 3^{-n}.
\]

(b) \[
\sum_{1 \leq i \leq n} 1 = n.
\]

(c) \[
\sum_{1 \leq i < j < k} 1 = \frac{n(n - 1)}{2}.
\]

(d) \[
\sum_{1 \leq i < j < k} 1 = \frac{n(n - 1)(n - 2)}{6}.
\]

(e) \[
\sum_{j=1}^{n} \sum_{k=j}^{n} \frac{1}{k^2} = H_n.
\]