Here's where Tuesday's (March 11) notes need to be fixed. We had
\[
\sum_{a \leq k < b} \int_k^{k+1} f(x) \, dx = \sum_{a \leq k < b} \left( \frac{f(k) + f(k + 1)}{2} - \int_k^{k+1} (\{x\} - \frac{1}{2}) f'(x) \, dx \right),
\]
whence we should have gotten
\[
\int_a^b f(x) \, dx = \sum_{a \leq k \leq b} f(k) - \frac{f(a) + f(b)}{2} - \int_a^b (\{x\} - \frac{1}{2}) f'(x) \, dx.
\]
Notice that \(b\) is included in the “\(a \leq k \leq b\)” under the Sigma. I think I failed to write this in class. Anyway, then our correct summation formula is
\[
\sum_{a \leq k \leq b} f(k) = \int_a^b f(x) \, dx + \frac{f(a) + f(b)}{2} + \int_a^b (\{x\} - \frac{1}{2}) f'(x) \, dx,
\]
and therefore the formula originally claimed for summing from 1 to \(n\) in class is correct:
\[
\sum_{k=1}^n f(k) = \int_1^n f(x) \, dx + \frac{f(n)}{2} + C_f - \int_n^\infty (\{x\} - \frac{1}{2}) f'(x) \, dx.
\]