Instructions: This homework is preparation for a closed-book examination for which you may, however, use one handwritten page of notes. You will be graded on how you derive your answers (except in short-answer problems), as well as the actual answers.

QUICKIES

(1) (36%) Determine numerical values or simple closed-form formulas for the given factorial powers, sums, and differences.

(a) \(10^3\)

(b) \(4^{-3}\)

(c) \(\sum_{k=0}^{10} [k^2 \text{ is odd}]\)

(d) \(\sum_{k=1}^{100} (4 + 3k)\)

(2) \(\sum_{5<k<n} k(k-1)(k-2)(k-3)(k-4)\)

(3) \(\sum_{0\leq j<n} \frac{1}{(j+1)(j+2)}\)

(h) \(\Delta 2^x\)

(i) \(\Delta H_x\)
(2) (4%) Write a recurrence (with initial condition) for $S_n$ that is equivalent to

$$S_n = \sum_{k=0}^{n} a_k.$$ 

(3) (20%) Let $\langle U_n \rangle$ be the sequence determined by the recurrence

$$U_n = 2U_{n-1} + 3 \quad \text{for} \quad n > 0$$

with the initial condition $U_0 = 4$. Prove by mathematical induction that

$$U_n = 7 \cdot 2^n - 3 \quad \text{for} \quad n \geq 0.$$
(4) (25%) Let $p(k) = k^3 + 4k^2 + 5k + 6$.

(a) Unfold/unwind/backtrack the recurrence

$$x_n = x_{n-1} + p(n) \quad \text{for} \quad n > 0$$

with initial condition $x_0 = 2003$ to get a formula for $x_n$ involving a summation.

(b) Evaluate $\sum_{k=1}^{n} p(k)$, i.e., evaluate the following sum in closed form as a function of $n$.

$$\sum_{k=1}^{n} (k^3 + 4k^2 + 5k + 6)$$

(As a precaution, you are advised to check your answer for small values of $n$.
Also, it is okay for a closed-form solution to involve factorial powers.)

(c) What is the solution $x_n$ of the recurrence in part (a)?
(5) (15%) Evaluate the following sum
\[ \sum_{k=1}^{n} \sum_{j=k}^{n} \frac{1}{2^j} \]
(a) numerically for \( n = 2 \) and \( n = 3 \);
(b) in closed form for all \( n \geq 2 \).