Problem Set 12

Solve Rosen section 7.2 problems 2, 4, 10, 24, 26.

Also solve the following problems.

Difference Equations ↔ Recurrence Relations

1. Rewrite the difference equation

$$\Delta^3 y_n - 5\Delta^2 y_n + 7\Delta y_n - 11 y_n = 0$$

   as a recurrence relation.

2. Rewrite the recurrence

$$y_n = 5y_{n-1} - 7y_{n-2} + 11y_{n-3}$$

   as a difference equation. (Notice that the result is not the same as in the first part.)

Gambler's Ruin

Suppose that a gambler repeatedly wagers on a chance event such as the color, red or black, that comes up on the spin of a roulette wheel. In the simple "gambler's ruin" model, it is assumed that the gambler always wagers the same amount—one dollar, say—and the gambler keeps placing bets until the gambler goes broke ("ruin") or achieves a goal amount (or "breaks the bank").

Let k be the number of dollars the gambler has; let T be the total number of dollars the gambler wants to have; let p be the probability that the gambler wins a dollar each time; let $q = 1 - p$ be the probability that the gambler loses a dollar; and let $r = q/p$ be the "odds against" the gambler. Let R_k be the probability that the gambler goes broke eventually, given that the gambler starts with \$$k$. Assume that the sequence of outcomes forms a Bernoulli trials process.

Then R_k satisfies the equation

$$R_k = pR_{k+1} + qR_{k-1}$$

for $0 < k < T$ and the boundary conditions $R_0 = 1$ and $R_T = 0$. Using the theory of linear recurrence relations with constant coefficients, find a nice formula for the solution, R_k, in terms of r. (There will be two cases: $r = 1$ and $r \neq 1$.) Extra credit: Explore and explain the practical consequences of this formula for a would-be gambler.