

Asymptotics Exercises

- (1) Arrange the following functions of n (sequences) via the relations \prec and \succ and \sim (assuming $n \rightarrow \infty$).

e^n
 H_n
 $\lg n$
 $\ln n$
 n^e
 $n!$
 $n \lg n$
 n^n

- (2) Beyond what value of x , approximately, will 1.01^x be larger than x^{100} ? What is the asymptotic relationship between 1.01^x and x^{100} ?

- (3) Express the following sums in the form $f(n) + O(g(n))$ where $f(n)$ is a logarithm, power, exponential, or other “simple” logarithmico-exponential function [GKP 442]

and $g(n) \prec f(n)$. Example: $\sum_{k=1}^n k^2 = \frac{1}{3}n^3 + O(n^2)$.

(a) $\sum_{k=1}^n k^5$

(b) $\sum_{k=0}^n p(k)$

where $p(k)$ is a polynomial of degree m : $p(k) = a_0 + a_1k + a_2k^2 + \cdots + a_mk^m$ and $a_m \neq 0$.

(c) $\sum_{k=0}^n 3^k$

(d) $\sum_{k=1}^n k^2 2^k$

(e) $\sum_{k=1}^n \frac{1}{k}$

REFERENCES

- [1] Ronald L. Graham, Donald E. Knuth, and Oren Patashnik, *Concrete Mathematics 2/e*, Addison-Wesley, Boston, 1994.