MCS 341 Review problems: discrete probability distributions

1. Identify the probability distribution that best fits each of the following random variables. Often it will be one of the distributions we studied. If it is not, try to recognize it as simply related to such a distribution.

   (a) W denotes the number of women in a random sample of 100 Gustavus students.
   $W \sim$ hypergeometric ($N = \#$ GAC students; $r = \#$ GAC women students; $n = 100$).

   (b) X is the number of particles detected by a Geiger counter in one hour.
   $X \sim$ Poisson ($\lambda =$ mean number arriving per hour)

   (c) Let Y be the number of heads one gets in repeatedly spinning a penny until one gets a tails.
   $Y = G - 1$ where $G \sim$ geometric ($p =$ 1-spin probability of tails)

   (d) Let Z represent the number of girl babies among the next 100 babies born in Mankato.
   $Z \sim$ binomial ($n = 100$; $p =$ probability of a girl baby)

   (e) Suppose that T tells us the number of rolls of a die it takes to get a "5" for the third time.
   $T \sim$ negative binomial ($r = 3$; $p = 1/6$)

   (f) Let U denote the number of elm seeds that fall in a certain square foot in a woods of elm trees that have escaped Dutch elm disease.
   $U \sim$ Poisson ($\lambda =$ mean number of seeds per square foot)

2. Suppose that a fair, balanced, cubic die with faces numbered 1 through 6 is rolled repeatedly. Let Y denote the number of rolls *preceding* the first roll to come up with a "6."

   (a) Give a formula for the probability function of Y.
   $P(Y = y) = (5/6)^y(1/6)$ for $y = 0, 1, 2, \ldots$ ($P(Y = y) = 0$ otherwise).

   (b) What is the probability that $Y < 5$?
   $P(Y < 5) = \sum_{y=0}^{4}(5/6)^y(1/6) = \frac{(5/6)^0-(5/6)^5}{1-5/6}(1/6) = 4561/7776 \approx .5981.$

   (c) Determine the expected value of Y.
   $G := Y + 1$ is a geometric r.v., so $E(Y) = E(G) - 1 = 1/p - 1 = 1/(1/6) - 1 = 5.$

   (d) Determine the variance of Y.
   $V(Y) = V(G) = q/p^2 = (5/6)/(1/6)^2 = 30.$

3. Suppose that the probability function $p(y)$ of the random variable Y is given by the following table.

y	-2	-1	0	1	2
$p(y)$	.30	.25	.10	.15	.20

(a) Calculate the expected value μ of Y.
$\mu = E(Y) = (-2)(.30) + (-1)(.25) + (0)(.10) + (1)(.15) + (2)(.20) = -.30.$

(b) Calculate the variance σ^2 of Y.
$E(Y^2) = (-2)^2(.30) + (-1)^2(.25) + (0)^2(.10) + (1)^2(.15) + (2)^2(.20) = 2.40.$
$\sigma^2 = V(Y) = E(Y^2) - \mu^2 = 2.40 - (-.30)^2 = 2.31.$
$\sigma = \sqrt{2.31} \approx 1.52.$

(c) What is the probability that $|Y - \mu| < 2\sigma$?
$P|Y - \mu| < 2\sigma) = P(|Y - (-.30)| < 2\sqrt{2.31}) = P(Y \in \{-2, -1, 0, 1, 2\}) = 1.$

(d) Let $Z = Y + 2$. What is the moment generating function of Z?
$m(t) = E(e^{tZ}) = .30 + .25e^t + .10e^{2t} + .15e^{3t} + .20e^{4t}.$

(e) What are the mean and variance of Z?
$E(Z) = E(Y) + 2 = -0.30 + 2 = 1.70.$
$V(Z) = V(Y) = 2.31.$