DIFFERENT KINDS OF MEANS
EXTRA CREDIT PROBLEM

A family of means based on the \textquotedbl{}p-norms\textquotedbl{} may be defined as follows (letting $p = r$ where $-1 \leq r \leq 1$): Assuming $x_1, \ldots, x_n > 0$, the $r^{th}$ mean is

$$
\left[ \frac{1}{n} \sum_{j=1}^{n} x_j^{r} / n \right]^{1/r}.
$$

For $r = 1$ this reduces to the arithmetic mean. For $r = -1$ it reduces to the harmonic mean. Prove that its limit, as $r \to 0$, is the geometric mean.