MCS 341 Extra Credit: The sum of a random number of r.v.s

Assume that $N, X_1, X_2, X_3, \ldots$ are independent random variables, that $N$ is nonnegative-integer valued, and that $X_1, X_2, X_3, \ldots$ are identically distributed. Let

$$\Phi_N(t) := E(t^N) = \sum_{n=0}^{\infty} P(N = n)t^n$$

be the probability-generating function for $N$ (see Section 3.10), and let

$$m(t) = m_{X_j}(t) = E(e^{tX_j})$$

be the moment-generating function for every $X_j$. Let

$$S_N = \sum_{j=1}^{N} X_j,$$

the sum of a random number of random variables. (Recall that $S_0 := 0$.) Let $m_{S_N}(t) = E(e^{tS_N})$, the moment-generating function of $S_N$

1. Prove that $m_{S_N}(t) = \Phi_N(m(t))$. Make sure that you cite reasons for each step of your calculation/proof.

2. Calculate the first and second derivatives of $m_{S_N}(t)$ and use the results to rederive the formulas for $E(S_N)$ and $V(S_N)$ obtained in class. (Review the theorems about the derivatives of probability-generating functions evaluated at 1 and the derivatives of moment-generating functions evaluate at 0.)