

Section: 18 or 8

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Holte, John M., Gustavus Adolphus College, St. Peter, MN, USA

Properties of O-regularly Varying Sequences: Elementary Proofs

Two sequences $f, g : \mathbb{N} \rightarrow \mathbb{R}^+$ are “asymptotically of the same order”—written $f(n) \asymp g(n)$ —if $f(n) = O(g(n))$ and $g(n) = O(f(n))$, and f is “O-regularly varying”—written $f \in OR_{seq}$ —if $f(\lfloor \lambda n \rfloor) \asymp f(n)$ for all positive λ . This paper provides proofs of major theorems about such sequences that are accessible to undergraduates. These theorems include a representation formula ($f \in OR_{seq}$ iff \exists bounded sequences g, h such that $f(n) = \exp \{g(n) + \sum_{k=1}^n h(k)/k\}$), a characterization of OR_{seq} analogous to the Bojanic-Seneta characterization of (ordinary) regularly varying sequences, and a discrete version of the OR Karamata Tauberian theorem/de Haan- Stadtmüller theorem.