When is $f(1) + \cdots + f(n)$ asymptotically of the same order as $nf(n)$?

John Holte

One frequently sees, e.g., in the analysis of algorithms, positive sequences $(f(n))$ for which $f(1)+\cdots+f(n)$ is asymptotically of the same order as $nf(n)$, i.e., their ratio is ultimately bounded away from 0 and ∞. If f is nondecreasing, a NASC is that $\{f(2n)/f(n)\}$ be bounded. For general positive sequences $(f(n))$, a NASC is that there exist sequences $(g(n))$ and $(h(n))$ bounded away from 0 and ∞ such that $f(n) = \{g(n)/n\} \exp \sum_{k=1}^{n} \{h(k)/k\}$. These and other results provide discrete parallels to the theory of R-O variation for functions of a real variable.