FRACTAL DIMENSION OF GENERALIZED BINOMIAL COEFFICIENTS MODULO A PRIME

JOHN M. HOLTE

ABSTRACT. Given a sequence (u_n) of positive integers generated by $u_1 = 1, u_2 = a, u_n = au_{n-1} + bu_{n-2} (n \geq 3)$, define the generalized factorial by $[n]! = u_1 u_2 \cdots u_n$ and the generalized binomial coefficient by $C(i,j) = [i+j]!/([i]![j]!)$. Assume that the prime p does not divide b. Let $r = \min\{n : p|u_n\}$. **Theorem 1 (Asymptotic abundance of residues):** $\#\{(i,j)|0 \leq i,j < rp^k \text{ and } C(i,j) \equiv \rho (\text{mod } p)\} \sim \frac{r(r+1)}{2(p-1)}\binom{p+1}{2}^k$ as $k \to \infty$ for $\rho = 1, \ldots, p-1$. **Theorem 2 (Fractal dimension):** Let $s_k = rp^k$. The Hausdorff dimension of $\cap_k \cup_{i,j<s_k} \{[i/s_k, (i+1)/s_k) \times [j/s_k, (j+1)/s_k) | p \text{ does not divide } C(i,j)\}$ is $\log \binom{p+1}{2} / \log p$.

GUSTAVUS ADOLPHUS COLLEGE, ST. PETER, MN 56082