FRACTAL DIMENSION OF GENERALIZED MULTINOMIAL COEFFICIENTS MODULO A PRIME

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Abstract. Given a sequence (u_n) of positive integers generated by $u_1 = 1, u_2 = a, u_n = au_{n-1} + bu_{n-2} (n \geq 3)$, define the generalized factorial by $[n]! = u_1 u_2 \cdots u_n$ and the generalized d-nomial coefficient by $C(n_1, \dots, n_d) = [n_1 + \cdots + n_d]!/([n_1]! \cdots [n_d]!)$. Assume that the prime p does not divide b. Let $r = \min\{n : p | u_n\}$.

Theorem 1 (Asymptotic abundance of residues): $\#\{(n_1, \dots, n_d) | 0 \leq n_1, \dots, n_d < rp^k$ and $C(n_1, \dots, n_d) \equiv \rho (\text{mod } p)\} \sim \frac{1}{p-1} \binom{r+d-1}{d} \binom{p+d-1}{d}^k$ as $k \to \infty$ for $\rho = 1, \dots, p-1$.

Theorem 2 (Fractal dimension): Let $s_k = rp^k$. The Hausdorff dimension of $\cap_k \cup \{[n_1/s_k, (n_1 + 1)/s_k) \times \cdots \times [n_d/s_k, (n_d+1)/s_k) | 0 \leq n_1, \dots, n_d < s_k, p$ does not divide $C(n_1, \dots, n_d)\}$ is $\log \binom{p+d-1}{d} / \log p$.

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