Global climate is determined by the radiation balance of the planet. There are three fundamental ways in which this balance can change:\(^1\):

- changing the incoming solar radiation (insolation)
- changing the albedo
- altering the Outgoing Longwave Radiation (OLR)

Anthropogenic interference with climate occurs first of all through a perturbation of the radiation balance (e.g., greenhouse effect, air pollution, land use change)\(^2\)

\(^1\) Intergovernmental Panel on Climate Change AR4 (2007), p. 465 (www.ipcc.ch)
\(^2\) M. Wild, Institute for Atmospheric and Climate Science, ETH, Zurich.

---

**Budyko’s EBM:**

\[
T = T(y, t) \quad y = \sin(\text{latitude})
\]

\[
R^\partial T / \partial t = Qs(y)(1 - \alpha(y)) - (A + BT) - C \left( T - \int_\eta^3 T \, dy \right)
\]

- \(R^\partial T / \partial t\): Net radiative heating
- \(Qs(y)\): Solar radiation
- \(\alpha(y)\): Albedo
- \(A\): Albedo correction
- \(BT\): Longwave radiation
- \(C\): Cloud feedback
- \(y\): Latitude
- \(T\): Temperature

Units: \(\text{W/m}^2 = \text{J/(s m}^2\)

Energy Balance and Greenhouse Gases

Budyko’s EBM: \( T = T(y, t) \quad y = \sin(\text{latitude}) \)

\[
R \frac{dT}{dt} = Q_S(y)(1 - \alpha(y)) - (A + BT) - C \left( T - \int_0^1 T dy \right)
\]

- \( A, B > 0 \)
- \( T \nRightarrow OLR \nRightarrow \)
- \( \text{CO}_2 \leftrightarrow A, B \ ? \)

IPCC Assessment Report 4 (2007), Working Group 1, p. 96

NASA: Earth Radiation Budget Experiment

Ray Pierrehumbert
University of Chicago

Principles of Planetary Climate
Cambridge, 2010

Chapter 3: Elementary models of radiation balance.
Chapter 4: Radiative transfer in temperature-stratified atmospheres


**Energy Balance and Greenhouse Gases**

**Radiation**: characterized by direction of propagation and frequency

- **Frequency** $\nu$ (Hz)
- **Wavelength** $\lambda$ (m)
- **Wavenumber** $n = \frac{\nu}{c} \text{ m}^{-1}$

$$\nu \lambda = c = 3 \times 10^8 \text{ m/s}$$

**Electromagnetic spectrum**

<table>
<thead>
<tr>
<th>Wavelength (m)</th>
<th>Wavenumber</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^0$</td>
<td>$0.001$</td>
<td>$3 \times 10^8$</td>
</tr>
<tr>
<td>$10^{-1}$</td>
<td>$0.01$</td>
<td>$3 \times 10^7$</td>
</tr>
<tr>
<td>$10^{-2}$</td>
<td>$0.1$</td>
<td>$3 \times 10^6$</td>
</tr>
<tr>
<td>$10^{-3}$</td>
<td>$1$</td>
<td>$3 \times 10^5$</td>
</tr>
<tr>
<td>$10^{-4}$</td>
<td>$10$</td>
<td>$3 \times 10^4$</td>
</tr>
<tr>
<td>$10^{-5}$</td>
<td>$100$</td>
<td>$3 \times 10^3$</td>
</tr>
<tr>
<td>$10^{-6}$</td>
<td>$1000$</td>
<td>$3 \times 10^2$</td>
</tr>
<tr>
<td>$10^{-7}$</td>
<td>$10000$</td>
<td>$3 \times 10^1$</td>
</tr>
<tr>
<td>$10^{-8}$</td>
<td>$100000$</td>
<td>$3 \times 10^0$</td>
</tr>
<tr>
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<td>$1000000$</td>
<td>$3 \times 10^{-1}$</td>
</tr>
<tr>
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<td>$10000000$</td>
<td>$3 \times 10^{-2}$</td>
</tr>
<tr>
<td>$10^{-11}$</td>
<td>$100000000$</td>
<td>$3 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

- **Radio**
- **Microwave**
- **Infrared**
- **Visible**
- **Ultra violet**
- **X-ray (soft)**
- **X-ray (hard)**
- **Gamma ray**

**Solid Angles and Steradians**

The measure in radians of an angle made by a collection of rays emanating from a point $P$ is the length of the arc of the unit circle centered on $P$ which the rays intersect.

The measure in steradians of a solid angle made by a collection of rays emanating from a point $P$ is the area of the patch of the unit sphere centered on $P$ which the rays intersect.

$$4\pi \text{ steradians}$$

**Energy of radiation**: measured by its intensity or radiance $I$

The amount of corresponding radiant energy is

$$dF_\nu = I_\nu \cos \theta \, d\omega \, dA \, dv \, dt$$

$I_\nu$ magnitude of radiant intensity $\text{W m}^{-2} \text{ Hz}^{-1} \text{ steradian}^{-1}$
Energy Balance and Greenhouse Gases

**Energy of radiation:** measured by its *intensity* or *radiance*

The amount of corresponding radiant energy is

\[ dF_\nu = I_\nu \cos \theta \, d\omega \, dA \, d\nu \, dt \]

Energy per unit frequency per unit area per unit time:

\[ dF_\nu = I_\nu \cos \theta \sin \theta \, d\theta \, d\phi \]

---

**Blackbody radiation:** radiation reacts so strongly with matter that it achieves thermodynamic equilibrium at the same temperature as the matter ("perfect absorbers and emitters").

Planck’s Function:

\[ I = B(\nu, T) = \frac{2\pi h^3}{c^2} \frac{1}{e^{\frac{h\nu}{kT}} - 1} \]

- \( k \): Boltzmann thermodynamic constant
- \( h \): Planck’s constant
- \( c \): speed of light

- frequency \( \nu \) \((Hz)\)
- wavelength \( \lambda \) \((m)\)
- wavenumber \( n = \nu/c \) \(m^{-1}\)

\( \nu \lambda = c = 3 \times 10^8 \text{ m/s} \)

**Stefan-Boltzmann Law:** total power exiting from each unit area of the surface of a blackbody:

\[ F = \int_0^\infty \pi B(\nu, T) \, d\nu = \sigma T^4 \quad \text{W m}^{-2} \]

\[ \sigma = 2\pi^5 k^4/(15c^2 h^3) \approx 5.67 \times 10^{-8} \text{ W m}^{-2} \text{K}^{-4} \]
Blackbody radiation

- **isotropic**: equally intense in all directions

\[ I = B(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{\hbar\nu/kT} - 1} \]

Total power exiting from each unit area of the surface of a blackbody:

\[ F = \int_0^\infty \pi B(\nu, T) \, d\nu = \sigma T^4 \]

Energy Balance and Greenhouse Gases

Radiation balance of planets: An idealized example

- the planetary albedo is spatially uniform.
- the planet radiates like a perfect blackbody (uniform surface temp. \( T \)).
- the planet’s atmosphere is perfectly transparent to the electromagnetic energy emitted by the surface.

\[ \begin{align*}
  a &= \text{planet's radius} \\
  T_* &= \text{star's temperature} \\
  r_* &= \text{star's radius} \\
  r &= \text{distance to star} \\
\end{align*} \]

- Total flux impinging on planet \( \sigma T_*^4/r^2 \) = solar constant \( L_* \)
- Energy absorption: \( \pi a^2 L_*(1 - \alpha) \)
- Energy loss: \( 4\pi a^3 \sigma T^4 \)

Equilibrate:

\[ \sigma T^4 = \frac{L_*}{4} (1 - \alpha) \]

\[ T = \frac{1}{\sqrt{2}} (1 - \alpha)^{1/4} \sqrt{\frac{T_*}{r}} T_* \]

\( (T_e = 255 \text{ K}) \)

Planet loses energy through emission at a lower wavenumber than that at which it receives energy from the star.

Flux absorption: Beer’s Law

Absorption is proportional to the flux \( F \) times the mass of the absorber along the path.

\[ dF = -\kappa_{abs} \rho_a F \, ds \]

\[ \kappa_{abs} = \kappa(\nu, p, T) \text{ m}^2/\text{kg} \]

\[ \rho_a \text{ density of the absorber} \]

Assumptions:

(i) no scattering: a parallel beam of radiation
(ii) plane-parallel approximation: atmospheric properties are functions only of the vertical coordinate

Greenhouse gas (GG):

Prevalence of clouds in the high, cold regions of the tropical atmosphere (condensed water small enough small enough to stay suspended for a long time.)

(Figure from PPC, p. 149)

The Earth’s observed zonal-mean OLR for January, 1986 (solid curve).

\( \sigma T^4_e \) (dashed curve).
Energy Balance and Greenhouse Gases

Greenhouse gas (GG)

Transmission spectra of major atmospheric gases


Energy Balance and Greenhouse Gases

Greenhouse gas (GG): Absorption coefficient and OLR spectrum. Toy example with one (fictitious) GG

Nitrogen N$_2$ -- 78.084%
Oxygen O$_2$ -- 20.9476%
Argon Ar -- 0.934%
Carbon Dioxide CO$_2$ -- 0.0314%
Neon Ne 0.001818%
Methane CH$_4$ -- 0.0002%
Ozone O$_3$ -- 0.000007%
Water vapor -- highly variable; typically makes up about 1%

This is composition of air in percent by volume, at sea level at 15°C and 101325 Pa. (CRC Handbook of Chemistry and Physics, edited by David R. Lide, 1997)
Energy Balance and Greenhouse Gases

Greenhouse gas: Absorption coefficient and OLR spectrum. Toy example with one (fictitious) GG

Absorption coefficient for pure CO$_2$ at $T = 293$ K, $p = 1000$ mb

Infrared emission peak for Earth centered on $\ell = 670$ cm$^{-1}$

The strength of the greenhouse effect is not so much a matter of how deep the ditch is, but how wide. (PPC, p. 217)

Rather than being unquestionably lethal, the results of a doubling of CO$_2$ may be merely catastrophic. --R. Pierrehumbert

Energy Balance and Greenhouse Gases

Greenhouse gases: Logarithmic dependence of OLR on CO₂

\[
(A + BT)
\]


\[
A = -352.08 + 9.56 \ln P; \quad B = 2.053 - 0.0514 \ln P; \quad P = pCO₂
\]


\[
A(\phi) = -326.4 + 9.161 \phi - 3.164 \phi^2 + 0.5486 \phi^3
\]

\[
B(\phi) = 1.953 - 0.04866 \phi + 0.01309 \phi^2 - 0.002577 \phi^3; \quad \phi = \ln(pCO₂/300)
\]


\[
R^2 \frac{dT}{dt} = S + \sum S_i \sin \left( \frac{2 \pi t}{T} \right) + g + k \ln \left( \frac{C(\theta)}{C_0} \right) - \sigma T^4
\]

\[
\frac{dC}{dt} = V - (W + W_t) + \beta(C_{\text{max}} - C)_t \max \left( \frac{dT}{dt} - c, 0 \right)
\]

**Equilibrium solutions**

\[
T^*(y, \eta) = \frac{Q}{B + C} \left( s(y)(1 - \alpha(y, \eta)) + \frac{C}{B}(1 - \bar{\alpha}(\eta)) \right) - \frac{A}{B} \bar{\alpha}(\eta) = \int_0^1 s(y)\alpha(y, \eta) dy
\]
Geological and paleomagnetic evidence indicate that during at least two Neoproterozoic glacial periods (~630 Ma and ~715 Ma) continental ice sheets flowed into the ocean near the equator.

Positive ice-albedo feedback; snowball Earth hypothesis

Alternative theory: thin strip of open ocean about the equator.

-- evidence that photosynthetic eukaryotes thrived both before and immediately after the Snowball Episodes.

-- evidence that multiple lineages of sponges may have survived these glaciations.

Energy Balance and Greenhouse Gases

The Jormungand Climate Model

\[ T_j(y, \eta) = \frac{Q}{B + C} \left( s(\eta)(1 - \alpha(\eta)) + \frac{C}{B}(1 - \bar{\alpha}(\eta)) \right) - \frac{A}{B} \]

\[ \bar{\alpha}(\eta) = \int_0^\eta s(y) \alpha(y, \eta) dy \]

Summary

- Lots of good science involved in the study of the atmosphere and its effects on climate. Involve physics and chemistry colleagues?
- Greenhouse gases and their effects on outgoing longwave radiation can be incorporated into conceptual climate models.
- Conceptual climate models are versatile: adjustments in the albedo function, for example, can lead to model behavior appropriate for different eras in the Earth’s past.
- Homework: Worksheet problems 2, 3, 4, 6, 7, 8, 13, 14

Thank You!