

# MCS 119 Midterm Exam Solutions

## Spring 2008

1. (12 pts) Compute the derivatives of the following functions. **Do not simplify.**

(a)  $y = x^{3/2} - 2x^{-3/2}$

**Solution:**  $y' = \frac{3}{2}x^{1/2} + 3x^{-5/2}$

(b)  $y = 4x\sqrt{x^2 + 4}$

**Solution:**  $y' = 4\sqrt{x^2 + 4} + 4x^2(x^2 + 4)^{-1/2}$

(c)  $y = \frac{x^2 - 2x + 2}{x^3 + 1}$

**Solution:**  $y' = \frac{(2x - 2)(x^3 + 1) - 3x^2(x^2 - 2x + 2)}{(x^3 + 1)^2}$

2. (12 pts) Find the equation of the tangent line to  $y^2 + xy - 2x = 7$  at the point  $(-2, 3)$ .

**Solution:** Using implicit differentiation we get

$$2y \frac{dy}{dx} + y + x \frac{dy}{dx} - 2 = 0.$$

Substituting  $x = -2$  and  $y = 3$  and solving for  $dy/dx$  gives  $dy/dx = -1/4$ . Thus the equation of the tangent line is

$$y - 3 = -\frac{1}{4}(x + 2).$$

3. (12 pts) BK is moving along the  $x$ -axis at a rate of 2 meters per second and TLo is moving along the  $y$ -axis at a rate of 10 meters per second. How fast is the distance between them increasing when BK is 5 meters from the origin and TLo is 12 meters from the origin?

**Solution:**  $x^2 + y^2 = h^2$  thus

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2h \frac{dh}{dt}.$$

$dx/dt = 2$  meters per second,  $dy/dt = 10$  meters per second,  $x = 5$  meters and  $y = 12$  meters. Using the pythagorean theorem we get that  $h = 13$  meters. Substituting these numbers into the derivative gives

$$dh/dt = 10 \text{ meters per second.}$$

4. (12 pts) Evaluate each of the following integrals.

(a)  $\int x^{3/2} - 2x^{-3/2} dx$

**Solution:**  $\frac{2}{5}x^{5/2} + 4x^{-1/2} + C$

(b)  $\int_0^2 (x+3)(x-4)dx$

**Solution:**

$$\left(\frac{x^3}{3} - \frac{x^2}{2} - 12x\right)\Big|_0^2 = -\frac{70}{3}$$

5. (12 pts) Use the method of substitution to evaluate the following integrals.

(a)  $\int 4x(x^4 + 7)^6 dx$

**Solution:** We did not grade this one because of a typo!

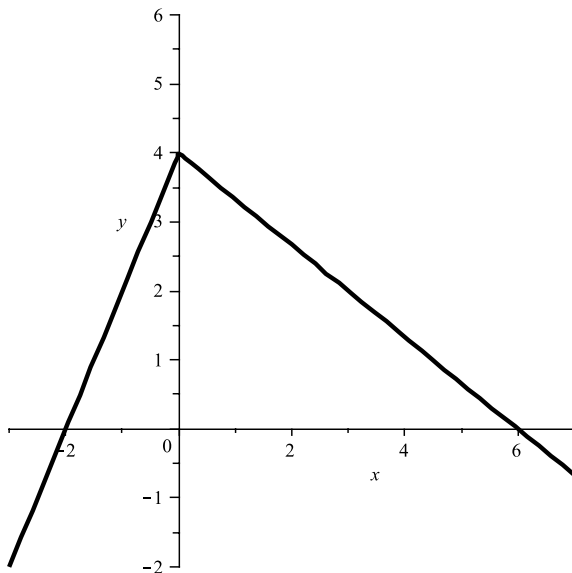
(b)  $\int (x+2)\sqrt{x^2+4x-3} dx$

**Solution:** Let  $u = x^2 + 4x - 3$  so that  $du = 2x + 4dx = 2(x+2)dx$ . This implies that  $\frac{1}{2}du = (x+2)dx$ . Using this we get

$$\frac{1}{2} \int u^{1/2} du = \frac{1}{3} u^{3/2} + C = \frac{1}{3} (x^2 + 4x - 3)^{3/2} + C.$$

6. (12 pts) Use the graph of  $y = f(x)$  in figure below to evaluate

$$\int_{-2}^6 f(x) dx.$$



**Solution:** The integral is interpreted as the area under the graph of  $y = f(x)$  and above the  $x$ -axis from  $x = -2$  to  $x = 6$ . This area is a triangle with a base of 8 units and a height of 4 units. Thus

$$\int_{-2}^6 f(x) dx = \frac{1}{2} \cdot 8 \cdot 4 = 16.$$

7. Suppose you have a function which has two x-intercepts, one at  $x = -3$  and one at  $x = 1$ . You also know that the derivative is positive for  $x < 0$  and negative when  $x > 0$ . Also the function only has one local extremum.

(a) Where is the local extremum of the graph of the function?

**Solution:** The local extremum is at  $x = 0$ .

(b) Does the graph have a local maximum or or a local minimum here?

**Solution:** The graph has a local maximum.

(c) Does the graph have an absolute maximum? If so at what  $x$ -value?

**Solution:** Yes the graph does have an absolute maximum at  $x = 0$ .

(d) Does the graph have an absolute minimum? If so at what  $x$ -value?

**Solution:** No the graph does not have an absolute minimum.

(e) Sketch the function.

It looks sort of like a parabola opening down.

8. (12 pts) The sum of 2 positive numbers is 24. Use calculus to find the values that maximize the first times the square of the second.

**Solution:** The helper equation is  $x + y = 24$  and the objective equation is  $P = xy^2$ . Solving the helper equation for  $x$  we get  $x = 24 - y$ . Substitute into the objective equation gives

$$P = (24 - y)y^2 = 24y^2 - y^3.$$

$$P' = 48y - 3y^2 = 3y(16 - y).$$

Thus the critical numbers are  $y = 0$  and  $y = 16$ . The second derivative test tells us that  $y = 16$  is a maximum. Using the helper equation we get that  $x = 24 - 16 = 8$  and thus the numbers are  $x = 8$  and  $y = 16$ .