4. Do exercise 2.18 on page 44.

Prove by induction that for every nonnegative integer n the following procedure computes $2n$:

```
(define f
 (lambda (n)
  (if (= n 0)
      0
      (+ 2 (f (- n 1))))))
```

Proof: We'll prove this by induction on n.

Base case: When $n = 0$, (f 0) returns to 0. Furthermore, 2(0) = 0, so (f 0) = 2(0).

Induction hypothesis: Assume that (f k) always returns $2k$ for all integers k such that $0 \leq k < n$.

Inductive step: Consider evaluating (f n) for some n > 0. Since n > 0, (f n) will return the value of (+ 2 (f (- n 1))), provided that (f (- n 1)) does returns something. Because (- n 1) evaluates to $(n - 1)$ and $0 \leq n - 1 < n$, we can therefore assume by our induction hypothesis that (f (- n 1)) does terminate, with the value $2(n - 1)$. Therefore, (+ 2 (f (- n 1))) evaluates to $2(n - 1) + 2$, and with some arithmetic work:

$$
\begin{aligned}
2(n-1) + 2 &= 2n - 2 + 2 \\
&= 2n
\end{aligned}
$$

Because $2(n-1) + 2 = 2n$, we see that (f n) does terminate with $2n$.

Conclusion: Therefore, by mathematical induction on n, (f n) terminates with the value $2n$ for every nonnegative integer n.

5. Do exercise 2.20 on pages 44-45.

Prove that the following procedure computes $n/(n + 1)$ for any nonnegative integer n. That is, (f n) computes $n/(n + 1)$ for any integer $n \geq 0$.

```
(define f
  (lambda (n)
    (if (= n 0)
        0
        (+ (f (- n 1))
           (/ 1 (* n (+ n 1)))))))
```

Proof: prove by induction on n.

Base case: when $n = 0$, (f 0) returns to 0. Furthermore, $0/(0 + 1) = 0$, so (f n) = $n/(n + 1)$.

Induction hypothesis: Assume that (f k) terminates with the value $k/(k + 1)$ for all k in the range $0 \leq k < n$.

Inductive step: Consider evaluating (f n), with $n > 0$. This will computes if evaluation of (f (- n 1)) does and will have the same value as (+ (f (- n 1)) (/ 1 (* n (+ n 1)))). Because (- n 1) evaluates to $n - 1$ and $0 \leq n - 1 < n$, we can assume by our induction hypothesis that (f (- n 1)) does terminate, with the value $(n - 1)/ ((n - 1) + 1)$. Therefore, (+ (f (- n 1)) (/ 1 (* n (+ n 1)))) evaluates to $[(n - 1)/ ((n - 1) + 1)] + [1/ (n^2 + n)]$, and with some arithmetic works:

$[(n - 1)/ ((n - 1) + 1)] + [1/ (n^2 + n)]$:

$$= [(n - 1)/ n] + [1/ (n^2 + n)]$$

$= [((n - 1)\ (n^2 + n))/\ (n\ (n^2 + n))] + [(1(n))/\ ((n^2 + n)\ (n)]$
$= [(n^3 + n^2 - n^2 - n)/\ (n^3 + n^2)] + [n/\ (n^3 + n^2)]$
$= [(n^3 - n + n)/\ (n^3 + n^2)]$
$= [n^3/\ (n^3 + n^2)]$
$= [n^3/\ (n^2\ (n + 1))]$
$= n/\ (n + 1)$

Thus, we see that (f n) does return the value $n/\ (n + 1)$. Therefore
(f n) = $n/\ (n + 1)$.

Conclusion: Therefore, by the mathematical induction on n, (f n) terminates with the value $n/\ (n + 1)$ for all $n \geq 0$.